MADE EASY & NEXT IAS GROUP

PRESENT

# MENNIT

NEET | IIT-JEE | FOUNDATION

**Corporate Office:** 44-A/1, Kalu Sarai, New Delhi 110016 | **Web:** www.mennit.com

# JEE (MAIN) 2022

## Test Date: 29th July 2022 (First Shift)

## PAPER-1

## Questions with Solutions

**Time : 3 Hours** **Maximum Marks: 300**

**Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.**

### IMPORTANT INSTRUCTIONS:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts**. **Part-A** is *Physics*, **Part-B** is *Chemistry* and Part-**C** is *Mathematics*. Each part has only two sections: Section-**A** and **Section-B**.
4. **Section – A :** Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.
7. **Section-B (1 – 10)** contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

# PART – A (PHYSICS)

## SECTION - A

### (One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

**Q.1** Given below are two statements : One is labelled as **Assertion (A)** and other is labeller as **Reason (R)**.

**Assertion (A) :** Time period of oscillation of a liquid drop depends on surface tension ($S$), if density of the liquid is $\rho$ and radius of the drop is $r$, then $T = K\sqrt{\rho r^3 / S^{3/2}}$ is dimensionally correct, where $K$ is dimensionless.

**Reason (R) :** Using dimensional analysis we get R.H.S. having different dimension than that of time period.

In the light of above statements, choose the correct answer from the options given below:

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(c) (A) is true but (R) is false

(d) (A) is false but (R) is true

**1.** **(d)**

$$T = k\sqrt{\frac{\rho r^3}{S^{3/2}}}$$

$$\downarrow \qquad \downarrow$$

$$[T] = \left[ML^{-3}\right]^{1/2}\left[L^3\right]^{1/2}\left[MLT^{-2}L^{-1}\right]^{\frac{-3}{4}}$$

$$\downarrow$$

$$[T] = \left[M^{1/2}L^{-3/2}L^{3/2}M^{-3/4}T^{3/2}\right]$$

$$= \left[M^{-\frac{1}{4}}\,T^{3/2}\right]$$

**Q.2** A ball is thrown up vertically with a certain velocity so that, it reaches a maximum height $h$. Find the ratio of the times in which it is at height $\frac{h}{3}$ while going up and coming down respectively.

(a) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (b) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

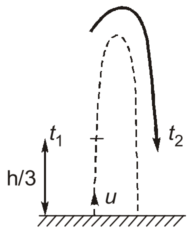
(c) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (d) $\frac{1}{3}$

**2.** **(b)**

$$h = \frac{u^2}{2g} \qquad \Rightarrow \qquad u = \sqrt{2gh}$$

$$\frac{h}{3} = \sqrt{2gh}\,t - 5t^2$$

$\Rightarrow \quad 5t^2 - \sqrt{20h}\,t + \frac{h}{3} = 0$




$$t = \frac{\sqrt{20h} \pm \sqrt{20h - 4\times 5\times \frac{h}{3}}}{10}$$

$$t = \frac{\sqrt{20h} \pm \sqrt{\frac{40}{3}h}}{10}$$

$$\therefore \quad \frac{t_1}{t_2} = \frac{\sqrt{20h} - \sqrt{\frac{40}{3}h}}{\sqrt{20h} + \sqrt{\frac{40}{3}h}} = \frac{1-\sqrt{\frac{2}{3}}}{1+\sqrt{\frac{2}{3}}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

**Q.3** If $t = \sqrt{x} + 4$, then $\left(\frac{dx}{dt}\right)_{t=4}$ is :

(a) 4 (b) zero

(c) 8 (d) 16

**3. (b)**

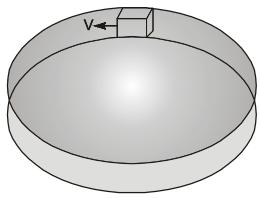
$$t = \sqrt{x} + 4$$

then $\frac{dt}{dx} = \frac{1}{2\sqrt{x}}$

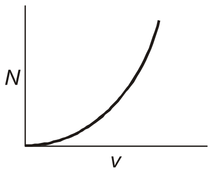
$\therefore \quad \left(\frac{dx}{dt}\right)_{t=4} = \left(2\sqrt{x}\right)_{t=4}$

$$= [2\,(t-4)]_{\text{at } t\,=\,4} = 0$$

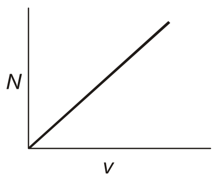
**Q.4** A smooth circular groove has a smooth vertical wall as shown in figure. A block of mass m moves against the wall with a speed $v$. Which of the following curve represents the correct relation between the normal reaction on the block by the wall ($N$) and speed of the block ($v$)?



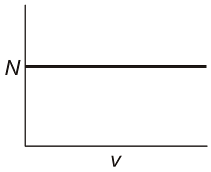

(a)



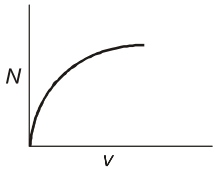

(b)




(c)

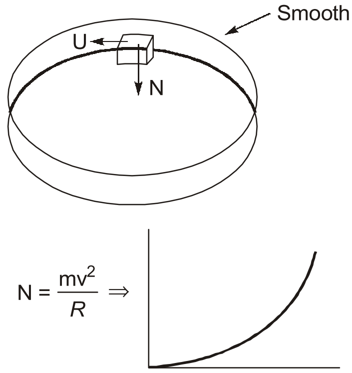



(d)



4. (a)

$$N = \frac{mv^2}{R}$$

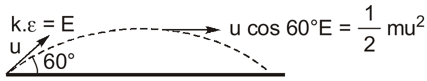



Q.5 A ball is projected with kinetic energy $E$, at an angle of 60° to the horizontal. The kinetic energy of this ball at the highest point of its flight will become :

(a) Zero (b) $\frac{E}{2}$

(c) $\frac{E}{4}$ (d) E

5. (c)




$$E = \frac{1}{2}mu^2$$

$$E_{\text{highest point}} = \frac{1}{2}m\left(\frac{u}{2}\right)^2$$

$$= \frac{1}{8}mu^2 = \frac{E}{4}$$

Q.6 Two bodies of mass 1kg and 3kg have position vectors $\hat{i}+2\hat{j}+\hat{k}$ and $-3\hat{i}-2\hat{j}+\hat{k}$ respectively. The magnitude of position vector of centre of mass of this system will be similar to the magnitude of vector :

(a) $\hat{i}+2\hat{j}+\hat{k}$ (b) $-3\hat{i}-2\hat{j}+\hat{k}$

(c) $-2\hat{i}-2\hat{k}$ (d) $-2\hat{i}-\hat{j}+2\hat{k}$

6. (a)

$$m_1 = 1 \text{ kg}$$

$$\vec{r}_1 = \hat{i}+2\hat{j}+\hat{k}$$

$$m_1 = 3 \text{ kg}$$

$$\vec{r}_2 = -3\hat{i}-2\hat{j}+\hat{k}$$

$$\vec{r}_{\text{com}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} = \frac{1}{4}(\hat{i}+2\hat{j}+\hat{k}+-9\hat{i}-6\hat{j}+3\hat{k})$$

$$= \frac{1}{4}(-8\hat{i}-4\hat{j}+4\hat{k})n$$

$$= -2\hat{i}-\hat{j}+\hat{k}$$

$$|\vec{r}_{\text{com}}| = \sqrt{4+1+1} = \sqrt{6}$$

**Q.7** Given below are two statements : One is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Clothes containing oil grease stains cannot be cleaned by water wash.

Reason (R) : Because the angle of contact between the oil/grease and water is obtuse.

In the light of the above statements, choose the correct answer from the option given below.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)

(c) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(c) (A) is true but (R) is false

(d) (A) is false but (R) is true

**7. (a)**

Water molecule doesn't stick with oil / grease drops due to obtuse angle of contact between them.

**Q.8** If the length of a wire is made double and radius is halved of its respective values. Then, the Young's modulus of the material of wire will :

(a) remain same
(b) become 8 times its initial value
(c) become $\frac{1}{4}^{th}$ of its initial value
(d) become 4 times its initial value

**8. (a)**

$Y$ is independent of dimensional parameter of a body.

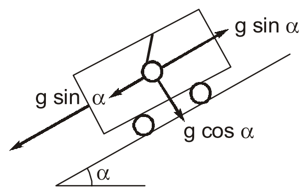
**Q.9** The time period of oscillation of a simple pendulum of length $L$ suspended from the roof of a vehicle, which moves without friction down an inclined plane of inclination $\alpha$, is given by :

(a) $2\pi\sqrt{L/(g\cos\alpha)}$
(b) $2\pi\sqrt{L/(g\sin\alpha)}$
(c) $2\pi\sqrt{L/g}$
(d) $2\pi\sqrt{L/g(\tan\alpha)}$

**9. (a)**

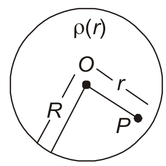
$$g_{\text{off}} = g\cos\alpha$$

$$\therefore \quad T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$




**Q.10** A spherically symmetric charge distribution is considered with charge density varying

$$\rho(r) = \begin{cases} \rho_0\left(\frac{3}{4} - \frac{r}{R}\right) & \text{for } r \le R \\ \text{Zero} & \text{for } r > R \end{cases}$$

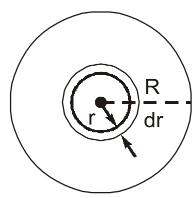



Where, $r(r < R)$ is the distance from the centre $O$ (as shown in figure). The electric field at point $P$ will be :

(a) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{3}{4} - \frac{r}{R}\right)$
(b) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{3}{4} - \frac{r}{R}\right)$
(c) $\frac{\rho_0 r}{4\varepsilon_0}\left(1 - \frac{r}{R}\right)$
(d) $\frac{\rho_0 r}{5\varepsilon_0}\left(1 - \frac{r}{R}\right)$

**10. (c)**

$$\rho_r = \begin{cases} \rho_0\left(\frac{3}{4} - \frac{r}{R}\right) & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$$



$$\phi_r = E\,4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = \int dq = \int_0^r \rho_0\left(\frac{3}{4}-\frac{r}{R}\right)4\pi r^2 dr$$

$$= 4\pi\rho_0\left[\frac{3}{4}\frac{r^3}{3}-\frac{r^4}{4R}\right]_0^r$$

$$= 4\pi\rho_0\left[\frac{r^3}{4}-\frac{r^4}{4R}\right]$$

$$= \pi\rho_0 r^3\left[1-\frac{r}{R}\right]$$

As,

$$E\,4\pi r^2 = \frac{\pi\rho_0 r^3}{\varepsilon_0}\left(1-\frac{r}{R}\right)$$

$$E = \frac{\rho_0 r}{4\varepsilon_0}\left(1-\frac{r}{R}\right)$$

**Q.11** Given below are two statements.

**Statement I:** Electric potential is constant within and at the surface of each conductor.

**Statement II :** Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point.

In the light of the above statements, choose the most appropriate answer from the options given below.

(a) Both statement I and statement II are correct

(b) Both statement I and statement II are incorrect

(c) Statement I is correct but statement II is incorrect

(d) Statement I is incorrect but statement II is correct

**11. (a)**

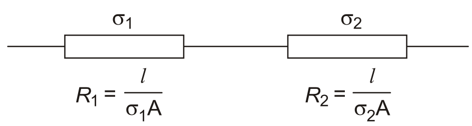
'*A*' at static equilibrium, $E_{(\text{inside conductor})} = 0$

**Q.12** Two metallic wires of identical dimensions are connected in series. If $\sigma_1$ and $\sigma_2$ are the conductivities of the these wires respectively, the effective conductivity of the combination is :

(a) $\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}$  (b) $\frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

(c) $\frac{\sigma_1+\sigma_2}{2\sigma_1\sigma_2}$  (d) $\frac{\sigma_1+\sigma_2}{\sigma_1\sigma_2}$

**12. (b)**




$$R = R_1 + R_2$$

$$\Rightarrow \quad \frac{2\ell}{\sigma A} = \frac{\ell}{\sigma_1 A}+\frac{\ell}{\sigma_2 A}$$

$$\Rightarrow \quad \frac{2}{\sigma} = \frac{1}{\sigma_1}+\frac{1}{\sigma_2}$$

$$\Rightarrow \quad \sigma = \frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$$

**Q.13** An alternating emf $E = 440 \sin 100\pi t$ is applied to a circuit containing an inductance of $\frac{\sqrt{2}}{\pi}H$. If an a.c. ammeter is connected in the circuit, its reading will be :

(a) 4.4 A (b) 1.55 A
(c) 2.2 A (d) 3.11 A

**13. (c)**

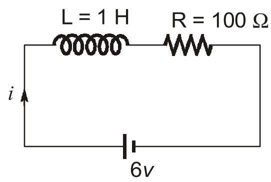
$$E = 440 \sin 100\pi t$$
$$\omega = 100\pi$$
$$L = \frac{\sqrt{2}}{\pi} \text{ H}$$
$$X_L = \omega L = 100\pi.\frac{\sqrt{2}}{\pi} = 100\sqrt{2}\,\Omega$$
$$i_{(rms)} = \frac{V_{(rms)}}{X_L} = \frac{440}{\sqrt{2}\times 100\sqrt{2}} = \frac{440}{200}$$
$$= \frac{220}{100} = 2.2 \text{ A}$$

**Q.14** A coil of inductance 1 H and resistance 100 Ω is connected to a battery of 6 V. Determine approximately :

1. The time elapsed before the current acquires half of its steady - state value.
2. The energy stored in the magnetic field associated with the coil at instant 15 ms after the circuit is switched on. (Given ln 2 = 0.693, $e^{-3/2} = 0.25$ )

(a) $t$ =10 ms; $U$ = 2 mJ (b) $t$ = 10 ms; $U$ = 1 mJ
(c) $t$ = 7 ms; $U$ = 1 mJ (d) $t$ = 7 ms; $U$ = 2 mJ

**14. (c)**




$$L = 1 \text{ H}$$
$$R = 100\ \Omega$$

As, $i = i_0 e^{-t/\tau}$.

For, $i = \frac{i_0}{2} \Rightarrow \frac{i_0}{2} = i_0 e^{-t/\tau}$

$\Rightarrow$ $-\ell n2 = -t/\tau$

$$t = \tau \ell n2 = \frac{L}{R}\ell n2. = \frac{1}{100}\ell n2 = 7 \text{ m/sec}$$

at $t$ = 15 ms

$$\tau = \frac{L}{R} = \frac{1}{100}$$
$$i = \frac{6}{100} e^{-\frac{15}{1/100}} = 0.6\, e^{-1500\times 10^{-3}}$$
$$= 0.06 \times 0.25 = 0.015 - \text{A}$$

So, $U = \frac{1}{2}Li^2 = \frac{1}{2}\times 1\times(15\times 10^{-3})^2$

$$= \frac{1}{2}(225)\times 10^{-6} = 112.5 \times 10^{-6} \text{ J} = 0.1125 \text{ mJ}$$

Q.15 Match List – I with List – II :

| | List – I | | List – II |
|---|---|---|---|
| (a) | UV rays | (i) | Diagnostic tool in medicine |
| (b) | X-rays | (ii) | Water purification |
| (c) | Microwave | (iii) | Communication, Radar |
| (d) | Infrared wave | (iv) | Improving visibility in foggy days |

Choose the correct answer from the options given below :

(a) (a) – (iii), (b) – (ii), (c) – (i), (d) – (iv)

(b) (a) – (ii), (b) – (i), (c) – (iii), (d) – (iv)

(c) (a) – (ii), (b) – (iv), (c) – (iii), (d) – (i)

(d) (a) – (iii), (b) – (i), (c) – (ii), (d) – (iv)

15. (b)

Based an theoretical data.

Q.16 The kinetic energy of emitted electron is E when the light incident on the metal has wavelength λ. To double the kinetic energy, the incident light must have wavelength :

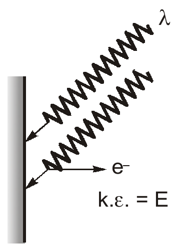
(a) $\frac{hc}{E\lambda - hc}$ (b) $\frac{hc\lambda}{E\lambda + hc}$

(c) $\frac{h\lambda}{E\lambda + hc}$ (d) $\frac{hc\lambda}{E\lambda - hc}$

16. (b)

$$\frac{hc}{\lambda} - \phi = E \quad ...(i)$$

$$\frac{hc}{\lambda'} - \phi = 2E \quad ...(ii)$$




$$hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) = E$$

$$\frac{1}{\lambda'} = \frac{E}{hc} + \frac{1}{\lambda}$$

$$\lambda' = \frac{hc\lambda}{E\lambda + hc}$$

Q.17 Find the ratio of energies of photos produced due to transition of an electron of hydrogen atom from its (i) second permitted energy level to the first level, and (ii) the highest permitted energy level to the first permitted level.

(a) 3 : 4 (b) 4 : 3

(c) 1 : 4 (d) 4 : 1

17. (a)

$$\Delta E_1 = -\frac{E_0}{4} + \frac{E_0}{1} = \frac{3}{4}E_0$$

$$\Delta E_2 = 0 - (E_0) = E_0$$

$$\frac{\Delta E_1}{\Delta E_2} = \frac{3}{4}$$

**Q.18** Find the modulation index of an AM wave having 8 V variation where maximum amplitude of the wave is 9 V.

(a) 0.8 (b) 0.5
(c) 0.2 (d) 0.1

**18. (a)**

Modulation index, $\mu = \frac{A_m}{A_c}$

$$\text{Variation} = 2A_m = 8$$

$$\Rightarrow \quad A_m = 4 \text{ v}$$

$$A_m + A_c = 9$$

$$\Rightarrow \quad A_c = 9 - A_m = 5 \text{ v}$$

$$\therefore \quad \mu = \frac{4}{5} = 0.8$$

**Q.19** A travelling microscope has 20 divisions per cm on the main scale while its vernier scale has total 50 divisions and 25 vernier scale divisions are equal to 24 main scale divisions, what is the least count of the travelling microscope?

(a) 0.001 cm (b) 0.002 mm
(c) 0.002 cm (d) 0.005 cm

**19. (c)**

MSD = 20 division per cm $\rightarrow$ 1 MSD = $\frac{1}{20}$ cm.

$$\text{VSD} = 50 \text{ divisions} \rightarrow$$

As, $$25 \text{ VSD} = 24 \text{ MSD}$$

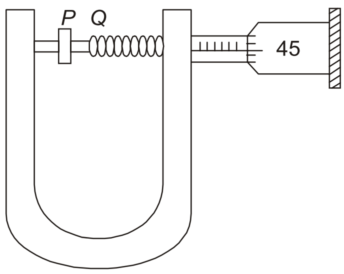
$$\therefore \quad 1 \text{ VSD} = \frac{24}{25} \text{ MSD}$$

$$\text{L.C.} = 1 \text{ MSD} - 1 \text{ VSD}$$

$$= 1 \text{ MSD} - \frac{24}{25} \text{ MSD} - \frac{1}{25} \text{ MSD} = \frac{1}{25} \times \frac{1}{20} \text{ cm.}$$

$$= \frac{1}{500} \text{ cm} = 0.002 \text{ cm}$$

**Q.20** In an experiment to find out diameter of wire using screw gauge, the following observations were noted :




(A) Screw moves 0.5 mm on main scale in one complete rotation.
(B) Total divisions on circular scale = 50
(C) Main scale reading is 2.5 mm
(D) 45th division of circular scale is in the pitch line
(E) Instrument has 0.03 mm negative error

Then the diameter of wire is :

(a) 2.92 mm (b) 2.54 mm
(c) 2.98 mm (d) 3.45 mm

20. (c)

$$\text{MS – Reading} = 2.5 \text{ mm}$$
$$50 \text{ division on CS} = 0.5 \text{ mm on M.S.}$$
$$\therefore \quad 45^{th} \text{ division on CS} = \frac{0.5}{50} \times 45 \text{ mm M.S.}$$
$$= \frac{0.5}{50} \times 45 \text{ mm M.S.}$$
$$= 0.5 \times 0.9 \text{ mm}$$
$$= 0.45 \text{ mm on M.S.}$$

So, $\text{Diameter} = 2.5 \text{ mm} + 0.45 \text{ mm} + 0.03 \text{ mm} = 2.98 \text{ mm}$

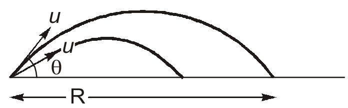
## SECTION - B

### (Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

**Q.1** An object is projected in the air with initial velocity u at an angle $\theta$. The projectile motion is such that the horizontal range *R*, is maximum. Another object is projected in the air with a horizontal range half of the range of first object. The initial velocity remains same in both the case. The value of the angle of projection, at which the second object is projected, will be _______degree.

1. (15 or 75)




$$R = \frac{u^2 \sin(2 \times 45°)}{g} = \frac{u^2}{g}$$
$$\frac{R}{2} = \frac{u^2}{2g} = \frac{u^2 \sin 2\theta}{g}$$
$$\sin 2\theta = \frac{1}{2}$$
$$\Rightarrow \quad 2\theta = 30°$$
$$\Rightarrow \quad \theta = 15°$$

**Q.2** If the acceleration due to gravity experienced by a point mass at a height h above the surface of earth is same as that of the acceleration due to gravity at a depth $\alpha h (h << R_e)$ from the earth surface. The value of $\alpha$ will be _______. (use $R_e$ = 6400 km)

2. (2)

$$g_{(\text{at } h)} = g_{(\text{at depth } \alpha h)} \qquad h << R$$
$$\Rightarrow \quad g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{\alpha h}{R}\right)$$
$$1 - \frac{2h}{R} = 1 - \frac{\alpha h}{R}$$
$$\Rightarrow \quad \frac{2h}{R} = \alpha \frac{h}{R}$$
$$\alpha = 2$$

**Q.3** The pressure $P_1$ and density $d_1$ of diatomic gas $\left(\gamma = \frac{7}{5}\right)$ changes suddenly to $P_2(>P_1)$ and $d_2$ respectively during an adiabatic process. The temperature of the gas increases and becomes _______ times of its initial temperature. (given $\frac{d_2}{d_1} = 32$).

**3. (4)**

For adiabatic process, $PV^\gamma = \text{constant}$

$\Rightarrow$ $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{d_2}{d_1}\right)^{\gamma-1} = (32)^{\left(\frac{7}{5}-1\right)} = (32)^{2/5}$$

$$= (2)^2 = 4$$

**Q.4** One mole of a monoatomic gas is mixed with three moles of a diatomic gas. The molecular specific heat of mixture at constant volume is $\frac{\alpha^2}{4}R$ J/mol K; then the value of $\alpha$ will be ______.

(Assume that the given diatomic gas has no vibrational mode.)

**4. (3)**

$n_1 = 1 \rightarrow$ monatomic

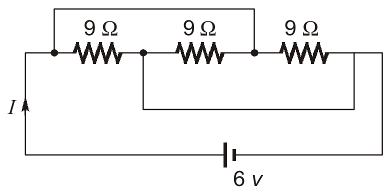
$n_2 = 3 \rightarrow$ diatomic

$$C_v = \frac{\alpha^2 R}{4} \text{ J/mol}^{-k}$$

As, $$C_{v(\text{mix})} = \frac{1\times\frac{3}{2}R + 3\times\frac{5}{2}R}{4} = \frac{9R}{4} = \frac{\alpha^2 R}{4}$$

$\therefore$ $\alpha = 3$

**Q.5** The current $I$ flowing through the given circuit will be ________ $A$.




**5. (2)**

$$i = \frac{V}{R_{\text{Net}}} = \frac{6}{3} = 2\text{ A}$$

**Q.6** A closely wounded circular coil of radius 5 cm produces a magnetic field of $37.68 \times 10^{-4}$ T at its centre. The current through the coil is _____ $A$. [Given, number of turns in the coil is 100 and $\pi = 3.14$]

**6. (3)**

$$B_{\text{centre}} = N\frac{\mu_0 i}{2r}$$

$\Rightarrow$ $$\frac{100\times4\pi\times10^{-7}\times i}{2\times5\times10^{-2}} = 37.68\times10^{-4}$$

$\Rightarrow$ $$4\pi\times10^{-4}\, i = 37.68\times10^{-4}$$

$\therefore$ $$i = \frac{37.68}{4\times3.14} = 3\text{A}$$

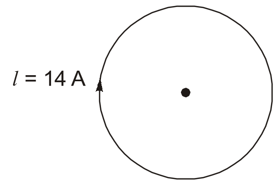
**Q.7** Two light beams of intensities $4I$ and $9I$ interfere on a screen. The phase difference between these beams on the screen at point $A$ is zero and at point $B$ is $\pi$. The difference of resultant intensities, at the point $A$ and $B$, will be __________ $I$.

7. (24)

$$I_A = 9I + 4I + 2\sqrt{9I \times 4I}\cos 0^\circ$$
$$= 13I + 12I = 25I$$
$$I_B = 9I + 4I + 2\sqrt{9I \times 4I}\cos\pi$$
$$= 13I - 12I = I$$
$$\therefore \quad I_A - I_B = 24I$$

**Q.8** A wire of length 314 cm carrying current of 14 A is bent to form a circle. The magnetic moment of the coil is ________ A – m$^2$. [Given $\pi$ = 3.14]

8. (11)

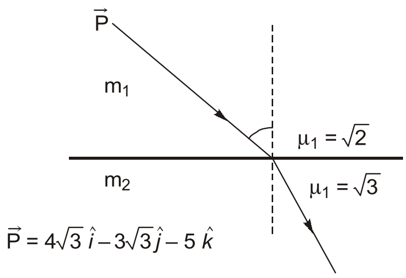



$$\ell = 2\pi r$$
$$\Rightarrow \quad 3.14\text{ cm} = 2 \times 3.14\, r$$
$$\Rightarrow \quad 3.14 = 2 \times 3.14\, r$$
$$\Rightarrow \quad r = \frac{1}{2}\text{m} = 0.5\text{ m}$$
$$\mu = iA$$
$$= 14 \times \pi r^2$$
$$= 14 \times 3.14 \times \frac{1}{4}$$
$$= 14^2 \times \frac{22}{7} \times \frac{1}{4} = 11$$

**Q.9** The *X-Y* plane be taken as the boundary between two transparent media $M_1$ and $M_2$. $M_1$ in $z \geq 0$ has a refractive index of $\sqrt{2}$ and $M_2$ with $Z < 0$ has refractive index of $\sqrt{3}$. A ray of light travelling in $M_1$ along the direction given by the vector $\vec{P} = 4\sqrt{3}\,\hat{i} - 3\sqrt{3}\,\hat{j} - 5\hat{k}$, is incident on the plane of separation. The value of difference between the angle of incident in $M_1$ and the angle of refraction in $M_2$ will be _______ degree.

9. (15)

For angle of incidence '$i$' :




$$\cos i = \frac{5}{\sqrt{48+27+25}} = \frac{5}{\sqrt{100}}$$
$$\Rightarrow \quad i = 60^\circ$$

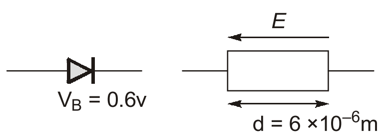
Using Snell's law:

$$\mu_1 \sin i = \mu_2 \sin r$$
$$\sin r = \frac{\sqrt{2}}{\sqrt{3}}\sin 60^\circ = \sqrt{\frac{2}{3}} \times \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}}$$
$$\therefore \quad r = 45^\circ$$

So, difference, $i - r = 60^\circ - 45^\circ = 15^\circ$

**Q.10** If the potential barrier across a p-n junction is 0.6 V. Then the electric field intensity, in the depletion region having the width of $6 \times 10^{-6}$ m, will be _______ $\times 10^5$ N/ C.

10. (1)




As, $Ed = V_B$

$$\therefore \quad E = \frac{V_B}{d} = \frac{0.6}{6 \times 10^{-6}} = 1 \times 10^5 \text{ N/C}$$

# PART – B (CHEMISTRY)

## SECTION - A

### (One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

**Q.1** Which of the following pair of molecules contain odd electron molecule and an expanded octet molecule?

(a) $BCl_3$ and $SF_6$ (b) NO and $H_2SO_4$

(c) $SF_6$ and $H_2SO_4$ (d) $BCl_3$ and NO

**1.** **(b)**

$BCl_3$ — electron deficient molecule

$SF_6$ — expanded octet molecule

NO — odd electron bond containing molecule

$H_2SO_4$ — expanded octet molecule

**Q.2** $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

20g 5g

Consider the above reaction, the limiting reagent of the reaction and number of moles of $NH_3$ formed respectively are:

(a) $H_2$, 1.42 moles (b) $H_2$, 0.71 moles

(c) $N_2$, 1.42 moles (d) $N_2$, 0.71 moles

**2.** **(c)**

| | $N_2(g)$ | + | $3H_2(g)$ | $\rightleftharpoons$ | $2NH_3(g)$ |
|---|---|---|---|---|---|
| t = 0 | $\frac{20}{28}$ | | $\frac{5}{2}$ | 0 | |

Here, $N_2$ is the limiting reagent

and No of moles of $NH_3$ = 2 × No of moles of $N_2$ reacted

$$= 2\times\frac{20}{28} = \frac{40}{28} = 1.42$$

**Q.3** 100 mL of 5% (w /v) solution of NaCl in water was prepared in 250 mL beaker. Albumin from the egg was poured into NaCl solution and stirred well. This resulted in a / an:

(a) Lyophilic sol (b) Lyophobic sol

(c) Emulsion (d) Precipitate

**3.** **(a)**

This is the method of preparation of lyophilic solution as condition is given.

**Q.4** The first ionization enthalpy of Na, Mg and Si, respectively, are : 496, 737 and 786 kJ $mol^{-1}$. The first ionization enthalpy ($kJmol^{-1}$) of Al is :

(a) 487 (b) 768

(c) 577 (d) 856

**4.** **(c)**

Following is the trends of I.E of given metals ($1^{st}$ I.P) Na < Al < Mg < Si

Q.5 In metallurgy the term "gangue" is used for:
(a) Contamination of undesired earthy material
(b) Contamination of metals, other than desired metal.
(c) Mineral which are naturally occurring in pure form
(d) Magnetic impurities in an ore.

5. (a)
In ores / mineral available earthy and undesired impurities are gangue.

Q.6 The reaction of zinc with excess of aqueous alkali, evolves hydrogen gas and gives:
(a) $Zn(OH)_2$ (B) ZnO
(c) $[Zn(OH)_4]^{2-}$ (d) $[ZnO_2]^{2-}$

6. (d)
Following is the reaction of Zn with excess alkali,

$$Zn + 2NaOH + 2H_2O \longrightarrow Na_2[Zn(OH)]_4 + H_2$$
$$\downarrow$$
$$Na_2ZnO_2 + 2H_2O$$
$$\downarrow$$
$$2Na^+ ZnO_2^{--}$$

Q.7 Lithium nitrate and sodium nitrate, when heated separately, respectively, give:
(a) $LiNO_2$ and $NaNO_2$ (b) $Li_2O$ and $Na_2O$
(c) $Li_2O$ and $NaNO_2$ (d) $LiNO_2$ and $Na_2O$

7. (c)

$$4LiNO_2 \xrightarrow[\Delta]{} 2Li_2O + 4NO_2 + O_2$$

$$NaNO_3 \xrightarrow[\Delta]{} NaNO_2 + \frac{1}{2}O_2$$

Q.8 Number of lone pairs of electrons in the central atom of $SCl_2$,$O_3$,$ClF_3$ and $SF_6$, respectively are:
(a) 0,1,2 and 2 (b) 2,1,2 and 0
(c) 1,2,2 and 0 (d) 2,1,0 and 2

8. (b)
$SCl_2$ — 2 lone pair of central atom
$O_3$ — 1 lone pair of central atom
$ClF_3$ — 2 lone pair of central atom
$SF_6$ — 0 lone pair of central atom

Q.9 In following pairs, the one in which both transition metal ions are colourless is:
(a) $Sc^{3+}$, $Zn^{2+}$ (b) $Ti^{4+}$, $Cu^{2+}$
(c) $V^{2+}$,$Ti^{3+}$ (d) $Zn^{2+}$, $Mn^{2+}$

9. (a)
$Sc^{3+}$ — $3d^0$ electron in last subshell
$Zn^{2+}$ — $3d^{10}$ electron in last subshell
$Ti^{4+}$ — $3d^0$ electron in last subshell
$Cu^{2+}$ — $3d^9$ electron in last subshell
$V^{2+}$ — $3d^3$ electron in last subshell
$Ti^{3+}$ — $3d^1$ electron in last subshell
$Mn^{2+}$ — $3d^5$ electron in last subshell

**Q.10** In neutral or faintly alkaline medium $KMnO_4$ being a powerful oxidant can oxidize, thiosulphate almost quantitatively, to sulphate. In this reaction overall change in oxidation state of manganese will be:

(a) 5 (b) 1
(c) 0 (d) 3

**10.** **(d)**

Following is the reaction of $KMnO_4$ in basic medium with thiosulphate anion.

$$8\overset{+7}{Mn}O_4^- + 3S_2O_3^{--} + H_2O \longrightarrow 8\overset{+7}{Mn}O_2 + 6SO_4^{--} + 2OH^-$$

Change = 7 – 4 = 3.

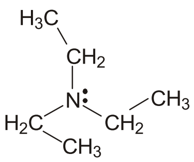
**Q.11** Which among the following pairs has only herbicides?

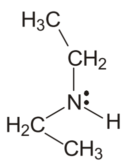
(a) Aldrin and Dieldrin (b) Sodium chlorate and Aldrin
(c) Sodium arsinate and Dieldrin (d) Sodium chlorate and sodium arsinite.

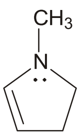
**11.** **(d)**

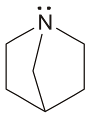
The pair of herbicides is sodium chlorate and sodium arsinite.

**Q.12** Which among the following is the strongest Bronsted base?

(a)


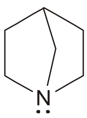

(b)



(c)



(d)


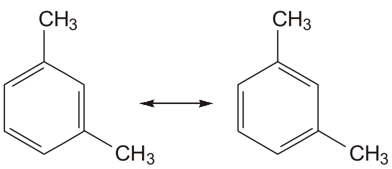

**12.** **(d)**

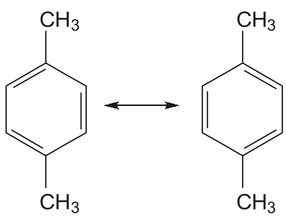
Due the absence of inversion and non resonating nature.

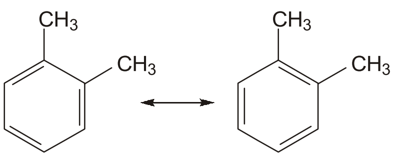


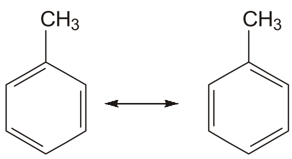
is the most basic in among all.

**Q.13** Which among the following pairs of the structure will give different products on ozonolysis? (Consider the double bonds in the structure are rigid and not delocalized.)

(a)


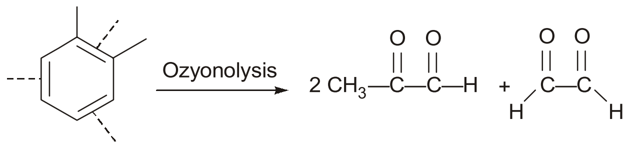

(b)


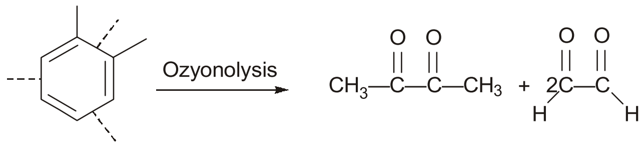

(c)



(d)


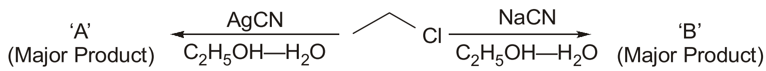
13. **(c)**

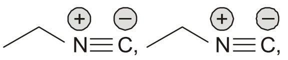
Total there product will produce by the ozonolysis of given structure

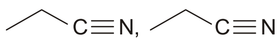


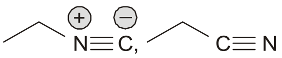



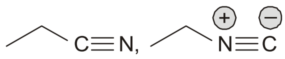

**Q.14** Considering the below reactions, the compound 'A' and compound 'B' respectively are:



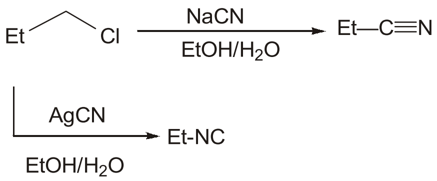

(a) 


(b) 


(c) 

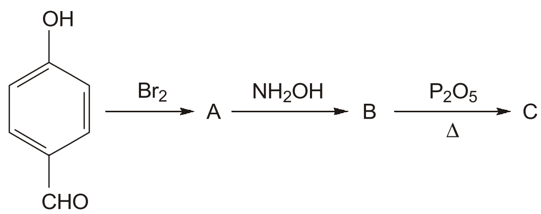

(d) 

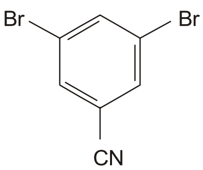

14. **(c)**

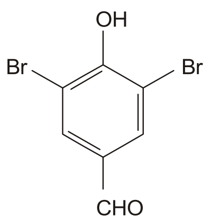


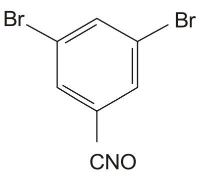

NaCN is ionisable salt where AgCN is non ionisable salt.

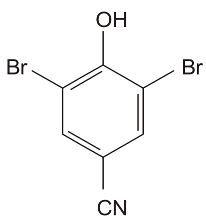
**Q.15** Consider the below reaction sequence, the Product 'C' is:



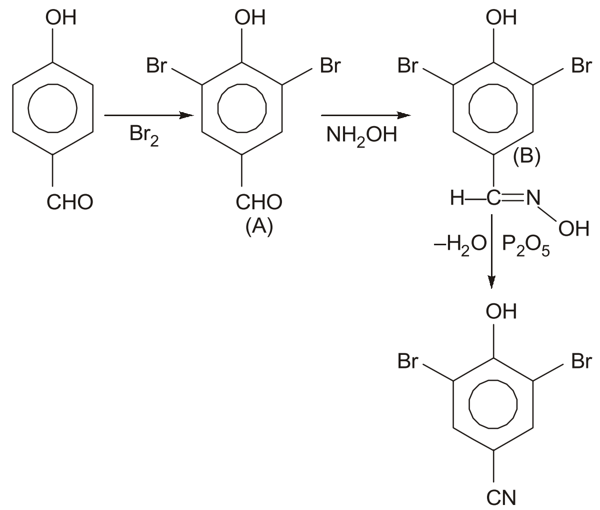

(a) 


(b) 

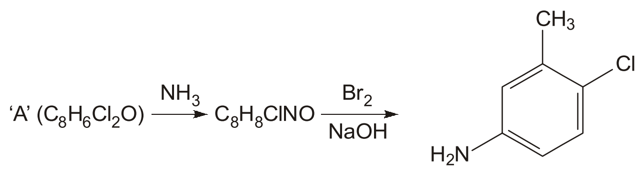

(c) 


(d) 

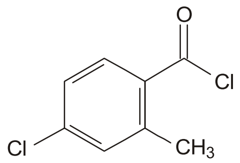
15. (d)



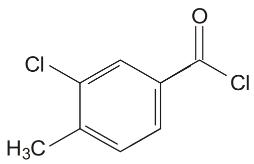

Q.16 Consider the below reaction, the compound 'A' is:



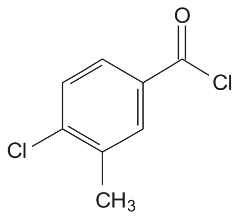

(a)



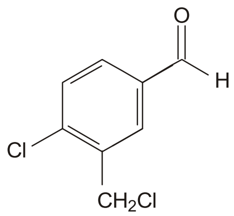
(b)



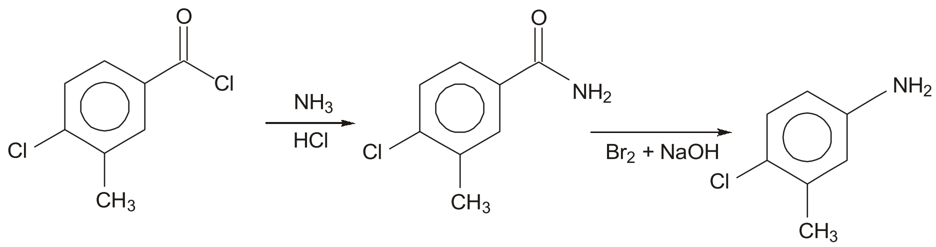
(c)



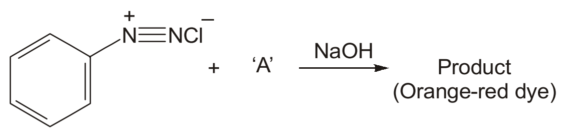
(d)

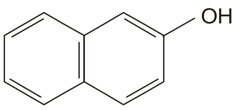


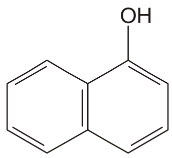
16. (c)

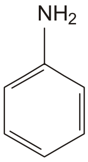


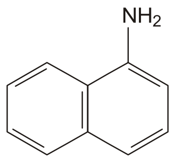

Q.17 Which among the following represent reagent 'A'?



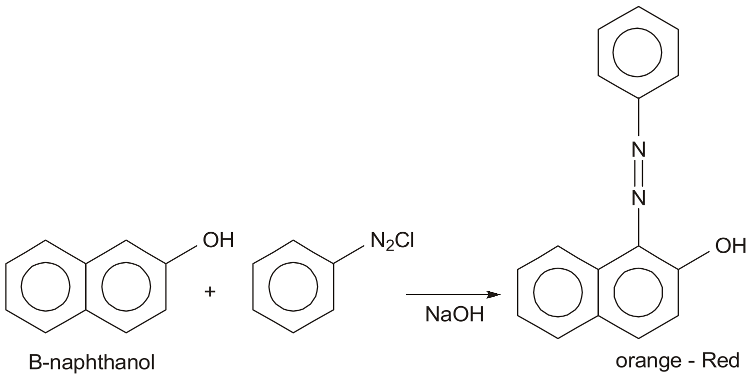
(a) 


(b) 

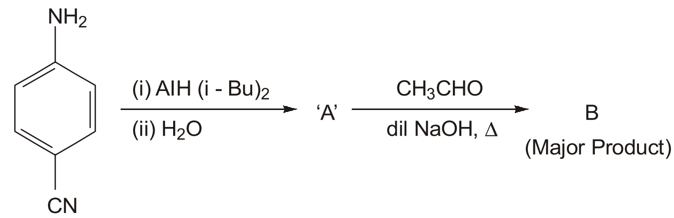

(c) 


(d) 

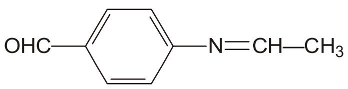

17. (a)

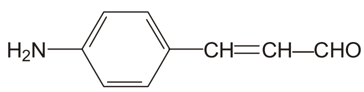


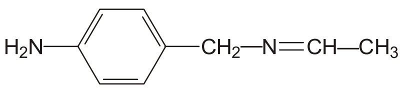

**Q.18** Consider the following reaction sequence:

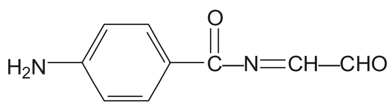



The product 'B' is:

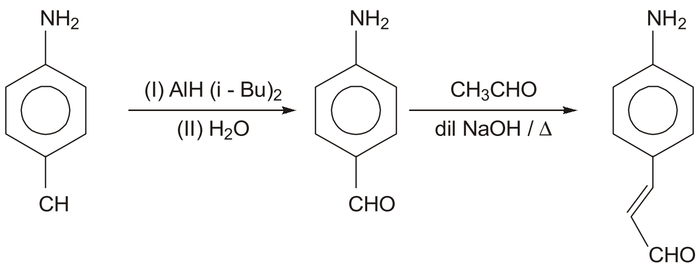
(a) 


(b) 


(c) 


(d) 


18. (b)

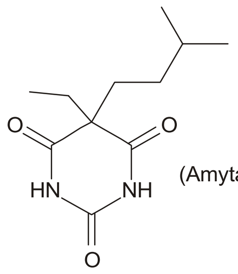



**Q.19** Which of the following compounds is an example of hypnotic drug?

(a) Seldane (b) Amytal

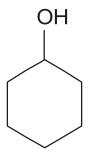
(c) Aspartame (d) Prontosil

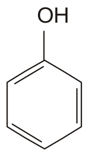
19. (b)

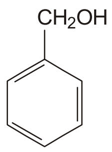


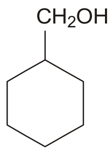
(Amytal)

Q.20 A compound 'X' is acidic and it is soluble in NaOH solution, but insoluble in $NaHCO_3$ solution. Compound 'X' also gives violet colour with neutral $FeCl_3$ solution. The compound 'X' is :

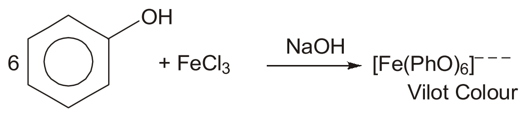
(a) 

(b) 

(c) 

(d) 

20. (b)



# SECTION - B

## (Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

**Q.1** Resistance of a conductivity cell (cell constant 129 $m^{-1}$) filled with 74.5 ppm solution of KCl is 100 Ω (labeled as solution 1). When the same cell is filled with KCl solution of 149 ppm, the resistance is 50 Ω (labeled as solution 2). The ratio of molar conductivity of solution 1 and solution 2 is i.e. $\frac{\wedge_1}{\wedge_2} = x \times 10^{-3}$. The value of $x$ is _____. (Nearest integer)
Given, molar mass of KCl is 74.5 g $mol^{-1}$.

1. **(1000)**

Given cell constant $\frac{\ell}{A} = 129\ m^{-1}$

Ratio of ppm of two solution of KCl = Ratio of moles per unit volume.

$$\frac{(ppm)_{KCl_1}}{(ppm)_{KCl_2}} = \frac{M_1}{M_2} = \frac{75.5}{149.0} = \frac{1}{2}$$

$$\frac{\wedge_1}{\wedge_2} = \frac{k_1 \times 1000 / M_1}{k_2 \times 1000 / M_2} = \frac{k_1}{k_2} \times \frac{M_2}{M_1} = \frac{R_2}{R_1} \times \frac{M_2}{M_1} = \frac{50}{100} \times \frac{2}{1} = 1$$

$$\therefore \quad \frac{\wedge_1}{\wedge_2} = 1000 \times 10^{-3}$$

$$\therefore \quad x = 1000 \times 10^{-3}$$

**Q.2** Ionic radii of cation $A^+$ and anion $B^-$ are 102 and 181 pm respectively. These ions are allowed to crystallize into an ionic solid. This crystal has cubic close packing for $B^-$. $A^+$ is present in all octahedral voids. The edge length of the unit cell of the crystal AB is _________ pm.(Nearest integer)

2. **(566)**

Edge length of the unit cell AB = a = $2(r_+ + r_-)$

$$\therefore \quad a = 2(102 + 181) = 2(283) = 566.$$

**Q.3** The minimum uncertainty in the speed of an electron in an one dimensional region of length $2a_0$ (Where $a_0$ = Bohr radius 52.9 pm) is__________ km $s^{-1}$.
(Given: Mass of electron = $9.1 \times 10^{-31}$ kg, Planck's constant $h = 6.63 \times 10^{-34}$ Js)

3. **(548)**

According to Heisenberg's uncertainly principle,

$$\Delta x \times \Delta v = \frac{h}{4\pi}$$

Or

$$\Delta x \times \Delta v = \frac{h}{4\pi \times m}$$

$$\therefore \quad \Delta v = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 2 \times 52.9 \times 10^{-12} \times 9.1 \times 10^{-31}} = 548273\ ms^{-1}$$

Or

$$\Delta v = 548\ km\ sec^{-1}$$

**Q.4** When 600 mL of 0.2 M $HNO_3$ is mixed with 400 mL of 0.1 M NaOH solution in a flask, the rise in temperature of the flask is __________ $\times 10^{-2}$ °C.
(Enthalpy of neutralization = 57 kJ $mol^{-1}$ and Specific heat of water = 4.2 $JK^{-1}$ $g^{-1}$)
(Neglect heat capacity of flask)

**4.** **(54)**

| | $HNO_3$ | + | NaOH | → | $NaNO_3$ | + | $N_2O$ |
|---|---|---|---|---|---|---|---|
| t = 0, | $(600 \times 0.2)_{meq}$ | | $(400 \times 0.1)_{meq}$ | | 40 meq | | 40 meq |
| | 120 meq | | 40 meq | | | | |
| t = ∞ | 80 meq | | ----------- | | | | |

Total heat produced during neutralization:

$$= \frac{40}{1000} \times 57 \times 1000\,J = 2280\text{ J} = m.s\theta$$

$2280 = 1000 \times 4.2 \times \theta$

$\therefore \quad \theta = 2280/4200 = 54.28 \times 10^{-2}$ C

**Q.5** If $O_2$ gas is bubbled through water at 303 K, the number of millimoles of $O_2$ gas that dissolve in 1 litre of water is _______________. (Nearest integer)
(Given: Henry's Law constant for $O_2$ at 303 K is 46.82 k bar and partial pressure of $O_2$ = 0.920 bar)
(Assume solubility of $O_2$ in water is too small, nearly negligible)

**5.** **(1)**
According to Henry's law

$$P = K^H \times \text{mole fraction of solute}$$

$$0.92 = 46.86 \times 1000 \times \frac{\text{mole of } O_2}{55.5}$$

$\therefore \quad$ Mole of $O_2 = 1.09 \times 10^{-3}$

$\therefore \quad$ Millimole $= 1.09 \approx 1$

**Q.6** If the solubility product of PbS is $8 \times 10^{-28}$, then the solubility of PbS in pure water at 298 K is $x \times 10^{-16}$ mol $L^{-1}$. The value of $x$ is__________. (Nearest integer) [Given : $\sqrt{2}$ = 1.41]

**6.** **(282)**

$$S = \sqrt{Ksp} = \sqrt{8 \times 10^{-28}} = 2.82 \times 10^{-14}$$

Or solubility of PbS = $282 \times 10^{-16}$

**Q.7** The reaction between X and Y is first order respect to X and zero order with respect to Y.

| Experiment | $\frac{[X]}{molL^{-1}}$ | $\frac{[Y]}{molL^{-1}}$ | $\frac{\text{Initial rate}}{molL^{-1}\,min^{-1}}$ |
|---|---|---|---|
| I | 0.1 | 0.1 | $2 \times 10^{-3}$ |
| II | L | 0.2 | $4 \times 10^{-3}$ |
| III | 0.4 | 0.4 | $M \times 10^{-3}$ |
| IV | 0.1 | 0.2 | $2 \times 10^{-3}$ |

Examine the data of table and calculate ratio of numerical values of M and L (Nearest integer)

**7.** **(40)**
According to condition
Rate $\propto [X]^1\ [Y]^0$
By using experimental data I and II

$$\frac{4\times10^{-3}}{2\times10^{-3}}=\frac{L}{0.1} \qquad \therefore \quad L=0.2$$

And by using experimental data I and II

$$\frac{M\times10^{-3}}{2\times10^{-3}}=\frac{0.4}{0.1}$$

$\therefore \quad M = 8$

Ratio of $\left(\frac{M}{L}\right)=\frac{8}{0.2}=40$

**Q.8** In a linear tetrapeptide (Constituted with different amino acids), (number of amino acids) – (number of peptide bonds) is _____.

8. **(1)**

No. of Amino acid units are 4 and

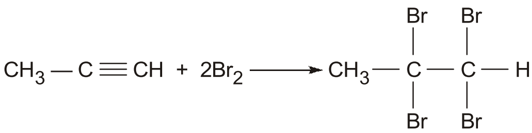
No. of peptide bonds are 3

In tetrapeptide

**Q.9** In bromination of Propyne, with Bromine 1,1,2,2-tetrabromopropane is obtained in 27% yield. The amount of 1,1,2,2- tetrabromopropane obtained from 1 g of Bromine in this reaction is _____ $\times 10^{-1}$ g. (Nearest integer) (Molar mass: Bromine = 80 g / mol)

9. **(3)**

Reaction is




Mass of product , 1,1, 2, 2- tetrobromopropane obtained

$$=\frac{1\times360}{160\times2}\times\frac{27}{100}=0.30375=3.0375\times10^{-1}$$

**Q.10** $[Fe(CN)_6]^{3-}$ should be an inner orbital complex. Ignoring the pairing energy, the value of crystal field stabilization energy for this complex is (–) _______ $\Delta_0$. (Nearest integer)

10. **(2)**

In complex $[Fe(CN)_6]^{---}$

$CN^-$ is the strong field ligand

$Fe^{3+} \rightarrow 3d^5\left[t_{2g}^5\, e_g^0\right]$

CFSE value = $5\times(0.4\,\Delta_0)=-2.0\,\Delta_0$

# PART – C (MATHEMATICS)

## SECTION - A

### (One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

**Q.1** Let $R$ be a relation from the set $\{1, 2, 3,....,60\}$ to itself such that $R\{(a, b) : b = pq\}$ where $p, q \geq 3$ are prime numbers. Then, the number of elements in $R$ is :

(a) 600 (b) 660

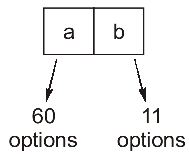
(c) 540 (d) 720

**1. (c)**

$a, b \in \{1, 2, 3,....,60\}$

$R = \{(a, b) : b = pq$, where $p, q \geq 3$ are prime$\}$

$\Rightarrow \quad 60 \times 11 = 660$



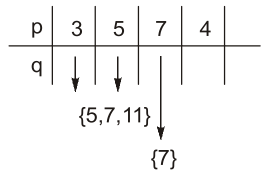

$p, q \in \{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59\} \rightarrow$ total 16

$b = pq \leq 60 \quad \begin{cases} p \leq 20 \Rightarrow p \leq 19 \\ q \leq 20 \Rightarrow q \leq 19 \end{cases}$ as $pq \leq 60$ and $\begin{cases} p \geq 3 \\ q \geq 3 \end{cases}$

$p,q \in \{3, 5, 7, 11, 13, 17, 19\}$

$\{3, 5, 7, 11, 13, 17, 19\}$

$7 + 3 + 1 = 11$




**Q.2** If $z = 2 + 3i$, then $z^5 + (\bar{z})^5$ is equal to:

(a) 244 (b) 224

(c) 245 (d) 265

**2. (a)**

$z^5 + (\bar{z})^5$

$= (2 + 3i)^5 + (2 - 3i)^5$

$= 2\,[{}^5C_0\, 2^5\, (3i)^0 + {}^5C_2\, 2^3 (3i)^2 + {}^5C_4\, 2^1 (3i)^4]$

$= 2\,(32 - 720 + 810) = 244$

**Q.3** Let $A$ and $B$ be two $3 \times 3$ non-zero real matrices such that $AB$ is a zero matrix. Then

(a) the system of linear equations $AX = 0$ has a unique solution

(b) the system of linear equations $AX = 0$ has infinitely many solutions

(c) $B$ is an invertible matrix

(d) adj($A$) is an invertible matrix

**3. (b)**

$A_{3\times3}$, $B_{3\times3}$ and $AB = 0$

$\neq 0 \quad \neq 0 \quad \Rightarrow |A| = 0$ and $|B| = 0$ only.

**Q.4** $\dfrac{1}{(20-a)(40-a)} + \dfrac{1}{(40-a)(60-a)} + \ldots + \dfrac{1}{(180-a)(200-a)} = \dfrac{1}{256}$, then the maximum value of a is :

(a) 198 (b) 202

(c) 212 (d) 218

4. (c)

$$\frac{1}{(20-a)(40-a)}+\frac{1}{(40-a)(60-a)}+\frac{1}{(60-a)(80-a)}+....+\frac{1}{(180-a)(200-a)}=\frac{1}{256}$$

$$\text{LHS}=\frac{1}{20}\left(\frac{1}{20-a}-\frac{1}{40-a}\right)+\frac{1}{20}\left(\frac{1}{40-a}-\frac{1}{60-a}\right)+...+\frac{1}{20}\left(\frac{1}{180-a}-\frac{1}{200-a}\right)$$

$$=\frac{1}{20}\left(\frac{1}{20-a}-\frac{1}{200-a}\right)=\frac{1}{20}\cdot\frac{180}{(20-a)(200-a)}$$

$$=\frac{9}{(20-a)(200-a)}=\frac{1}{256} \quad \text{(given)}$$

$\Rightarrow \quad a^2 - 220a + 4000 - 2304 = 0 \quad \Rightarrow \quad a^2 - 220a + 1696 = 0$

$D = 220^2 - 4 \times 1696 = 16\,(552 - 424) = 16 \times 2601$

$\sqrt{D} = 4 \times 51 = 204$

$a = \dfrac{220 \pm 204}{8} = 212^2$

Q.5 If $\lim\limits_{x\to 0}\dfrac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}=\dfrac{2}{3}$, where $\alpha, \beta, \gamma \in R$, then which of the following is NOT correct?

(a) $\alpha^2 + \beta^2 + \gamma^2 = 6$

(b) $\alpha\beta + \beta\gamma + \gamma\alpha + 1 = 0$

(c) $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 0$

(d) $\alpha^2 - \beta^2 + \gamma^2 = 4$

5. (c)

$$\lim_{x\to 0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}$$

$$=\lim_{x\to 0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x^3}$$

$$=\lim_{x\to 0}\frac{\alpha\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+...\right)+\beta\left(1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+...\right)+\gamma\left(x-\frac{x^3}{3!}+...\right)}{x^3}$$

$$=\lim_{x\to 0}\frac{(\alpha+\beta)+(\alpha-\beta+\gamma)x+\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)x^2+\left(\frac{\alpha}{6}-\frac{\beta}{6}-\frac{\gamma}{6}\right)x^3+....}{x^3}=\frac{2}{3} \quad \text{(given)}$$

$$=\alpha+\beta=0, \alpha-\beta+\gamma=0, \frac{\alpha+\beta}{2}=0, \frac{\alpha}{6}-\frac{\beta}{6}-\frac{\gamma}{6}=\frac{2}{3}$$

$\Rightarrow \beta = -\alpha \quad \gamma = -2\alpha \qquad \alpha - \beta - \gamma = 4 \qquad \Rightarrow \quad \alpha + \alpha + 2\alpha = 4 \qquad \Rightarrow \gamma = 1$

$\alpha = 1, \quad \beta = -1, \quad \gamma = -2$

Q.6 The integral $\displaystyle\int_0^{\frac{\pi}{2}}\frac{1}{3+2\sin x+\cos x}dx$ is equal to:

(a) $\tan^{-1}(2)$

(b) $\tan^{-1}(2)-\dfrac{\pi}{4}$

(c) $\dfrac{1}{2}\tan^{-1}(2)-\dfrac{\pi}{8}$

(d) $\dfrac{1}{2}$

6. (b)

$$\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\sin x+\cos x}$$

Put $\tan\frac{x}{2} = t$

$\Rightarrow$ $\frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$$= \int_{t=0}^{t=1} \frac{2dt/1+t^2}{3+\frac{4t}{1+t^2}+\frac{1-t^2}{1+t^2}} = \int_0^1 \frac{2dt}{3\left(1+t^2\right)+4t+1-t^2}$$

$$= \int_0^1 \frac{2dt}{2t^2+4t+4} = \int_0^1 \frac{dt}{(t+1)^2+1} = \left[\tan^{-1}(t+1)\right]_0^1 = \tan^{-1}2-\tan^{-1}1$$

Q.7 Let the solution curve $y = y(x)$ of the differential equation $(1 + e^{2x})\left(\frac{dy}{dx}+y\right)=1$ pass through the point $\left(0,\frac{\pi}{2}\right)$. Then $\lim\limits_{x\to\infty} e^x y(x)$ is equal to:

(a) $\frac{\pi}{4}$

(b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$

(d) $\frac{3\pi}{2}$

7. (b)

$$\frac{dy}{dx}+y = \frac{1}{1+e^{2x}}$$

$$\text{IF} = e^x$$

$$ye^x = \int\frac{e^x}{1+e^{2x}}dx$$

$$= \int\frac{dt}{1+t^2}$$

$$= \tan^{-1}(t) + c$$

$$ye^x = \tan^{-1}(e^x) + c$$

Point $\left(0,\frac{\pi}{2}\right)$ $\Rightarrow$ $\frac{\pi}{2} = \frac{\pi}{4}+c$

$$c = \frac{\pi}{4}$$

$$y = e^{-x}\tan^{-1}(e^x)+\frac{\pi}{4}e^{-x}$$

$$\lim_{x\to\infty} e^x y = \lim_{x\to\infty}\tan^{-1}e^x+\frac{\pi}{4} = \frac{3\pi}{4}$$

Q.8 Let a line L pass through the point of intersection of the lines $bx + 10y - 8 = 0$ and $2x - 3y = 0$, $b \in R - \left\{\frac{4}{3}\right\}$. If the line $L$ also passes through the point (1, 1) and touches the circle $17(x^2 + y^2) = 16$, then the eccentricity of the ellipse $\frac{x^2}{5} + \frac{y^2}{b^2} = 1$ is:

(a) $\frac{2}{\sqrt{5}}$

(b) $\sqrt{\frac{3}{5}}$

(c) $\frac{1}{\sqrt{5}}$

(d) $\sqrt{\frac{2}{5}}$

8. (b)

$$y - 1 = m(x - 1)$$

$$mx - y + 1 - m = 0$$

$$\frac{|1-m|}{\sqrt{m^2+1}} = \frac{4}{\sqrt{17}}$$

$$17(1 - m)^2 = 16(m^2 + 1)$$

$$\Rightarrow \quad m^2 - 3m + 1 = 0$$

$$\left.\begin{array}{r} bx + 10y - 8 = 0 \\ y = \frac{2}{3}x \end{array}\right\} \Rightarrow \left(b + \frac{20}{3}\right)x = 8$$

$$x = \frac{24}{3b+20}$$

$$y = \frac{16}{3b+20}$$

$$L : y - 1 = (17 \pm 12\sqrt{2})(x - 1)$$

$$\Rightarrow \quad \frac{-4-3b}{3b+20} = 17 \pm 12\sqrt{2})\left(\frac{4-3b}{3b+20}\right)$$

$$\Rightarrow \quad \frac{3b+4}{3b-4} = 17 \pm 12\sqrt{2}$$

$$(+) \rightarrow \frac{6b}{8} = \frac{18 \pm 12\sqrt{2}}{16 \pm 12\sqrt{2}}$$

$$\Rightarrow \quad \frac{3b}{4} = \frac{9 \pm 6\sqrt{2}}{8 \pm 6\sqrt{2}}$$

$$b = \frac{4}{3} \cdot \frac{9+6\sqrt{2}}{8+6\sqrt{2}} \quad \text{or} \quad b = \frac{4}{3} \cdot \frac{9-6\sqrt{2}}{8-6\sqrt{2}}$$

$$= \frac{2(3+2\sqrt{2})}{4+3\sqrt{2}} \quad \text{or} \quad b = \frac{2(3-2\sqrt{2})}{4-3\sqrt{2}}$$

$$b = \frac{2(3+2\sqrt{2})(4-3\sqrt{2})}{-2}$$

$$= \frac{12+8\sqrt{2}-9\sqrt{2}-12}{-1} = \sqrt{2}$$

$$b = \frac{2(3-2\sqrt{2})(4+3\sqrt{2})}{-2}$$

$$= \frac{12-8\sqrt{2}+9\sqrt{2}}{-1} = -\sqrt{2}$$

$$b^2 = 2$$

$$\frac{x^2}{5}+\frac{y^2}{2} = 1$$

$$2 = 5(1-e^2)$$

$$e^2 = \frac{3}{5}$$

$$e = \sqrt{\frac{3}{5}}$$

**Q.9** If the foot of the perpendicular from the point $A(-1, 4, 3)$ on the plane $P: 2x + my + nz = 4$, is $\left(-2,\frac{7}{2},\frac{3}{2}\right)$, then the distance of the point $A$ from the plane $P$, measrued parallel to a line with direction ratio 3, –1, –4, is equal to:

(a) 1 (b) $\sqrt{26}$

(c) $2\sqrt{2}$ (d) $\sqrt{14}$

**9.** **(b)**

$\left(-2,\frac{7}{2},\frac{3}{2}\right).M$

$A(-1, 4, 3)$

$$2x + my + nz = 4$$

$$M \Rightarrow -4+\frac{7m}{2}+\frac{3n}{2} = 4$$

$$7m + 3n = 16$$

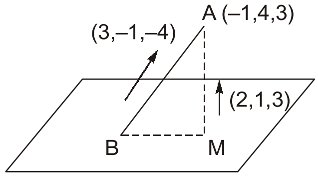
$$\overrightarrow{AM} = \left(-1,\frac{-1}{2},\frac{-3}{2}\right)$$

$$\frac{2}{-1} = \frac{m}{\frac{-1}{2}} = \frac{n}{\frac{-3}{2}}$$

$\Rightarrow$ $m = 1$

$\Rightarrow$ $n = 3$

**Plane :** $2x + y + 3z = 4$




$$\cos\theta = \frac{|6-1-12|}{\sqrt{26}\sqrt{14}} = \frac{7}{2\sqrt{91}}$$

$$AM = \frac{|-2+4+9-4|}{\sqrt{14}} = \frac{7}{\sqrt{14}}$$

$$\cos\theta = \frac{AM}{AB}$$

$\Rightarrow$
$$AB = \frac{7/\sqrt{14}}{7/2\sqrt{91}} = \frac{2\sqrt{91}}{\sqrt{14}} = 2\sqrt{\frac{13\times7}{2\times7}} = \sqrt{26}$$

**Q.10** Let $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. Let $\vec{c}$ be a vector satisfying $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda\vec{c}$. If $\vec{b}$ and $\vec{c}$ are non-parallel, then the value of $\lambda$ is:

(a) –5 (b) 5
(c) 1 (d) –1

**10. (a)**

$$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$$

$$= \vec{b} + \lambda\vec{c} \quad \text{(given)}$$

$$\vec{a}\cdot\vec{c} = 1$$

$$\lambda = -\vec{a}\cdot\vec{b}$$

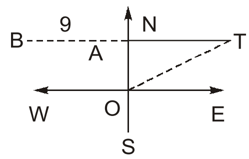
$$\lambda = -(3, 1, 0) . (1, 2, 1)$$

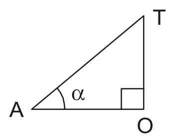
$$= -(3 + 2) = -5$$

**Q.11** The angle of elevation of the top of a tower from a point A due north of it is $\alpha$ and from a point $B$ at a distance of 9 units due west of $A$ is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point B from the tower is 15 units, then $\cot\alpha$ is equal to :

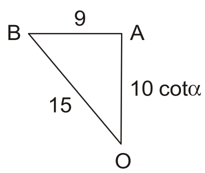
(a) $\frac{6}{5}$ (b) $\frac{9}{5}$
(c) $\frac{4}{3}$ (d) $\frac{7}{3}$

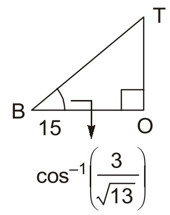
**11. (a)**




AB = 9




$$100\cot^2\alpha = 15^2 - 9^2 = 144$$

$$\frac{OT}{15} = \frac{2}{3}$$

$$\Rightarrow \quad \cot\alpha = \frac{6}{5}$$

$$OT = 10$$

$\Delta AOT$, $\quad OA = OT\cos\alpha = 10\cot\alpha$

**Q.12** The statement $(p \wedge q) \Rightarrow (p \wedge r)$ is equivalent to :

(a) $q \Rightarrow (p \wedge r)$ (b) $p \Rightarrow (p \wedge r)$
(c) $(p \wedge r) \Rightarrow (p \wedge q)$ (d) $(p \wedge q) \Rightarrow r$

**12. (d)**

$(p \wedge q) \rightarrow (p \wedge r)$

$\equiv \sim (p \wedge q) \vee (p \vee r)$

(a) $(\sim q) \vee (p \wedge r)$ (b) $(\sim p) \vee (p \wedge r)$
(c) $\sim (p \wedge r) \vee (p \wedge q)$ (d) $\sim (p \wedge q) \vee r$

Option (d) is correct.

**Q.13** Let the circumcentre of a triangle with vertices $A(a, 3)$, $B(b, 5)$ and $C(a, b)$, $ab > 0$ be $P(1, 1)$. If the line $AP$ intersects the line $BC$ at the point $Q(k_1, k_2)$, then $k_1 + k_2$ is equal to :

(a) 2 (b) $\frac{4}{7}$
(c) $\frac{2}{7}$ (d) 4

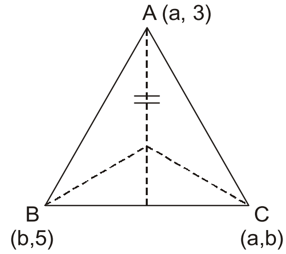
13. (b)

$$(a-1)^2 + 4 = (b-1)^2 + 16$$
$$= (a-1)^2 + (b-1)^2$$
$$ab > 0$$
$$(b-1)^2 = 4$$

and $(a-1)^2 = 16$

$\Rightarrow$ $b = 1 \pm 2 = 3, -1$

$$a = 1 \pm 4 = 5, -3$$




For $a = 5,\ \left.\begin{matrix} A(5,3) \\ B(3,5) \\ C(5,3) \end{matrix}\right]$ (not possible)

For $a = 3,\ b = 1\ \left\{\begin{matrix} A(-3,3) \\ B(-1,5) \\ C(-3,-1) \end{matrix}\right.$ (possible)

Line $AP$: $y - 1 = \frac{2}{-4}(x-1)$

$$y - 1 = -\frac{1}{2}(x-1)$$

$$x + 2y = 3 \quad \text{...(i)}$$

Line $BC$: $B(-1, 5)$, $C(-3, -1)$

$$y - 5 = \frac{6}{2}(x-1)$$

$$3x - y = -8 \quad \text{...(ii)}$$

$$x + 2y = 3$$

$$3x - y = -83 \times 2$$

$$\oplus \rightarrow 7x = -13$$

$\Rightarrow$ $x = \frac{-13}{7}$

$$y = \frac{-39}{7} + 8 = \frac{17}{7}$$

$$k_1 + k_2 = x + y = \frac{4}{7}$$

**Q.14** Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{4}$. If $\theta$ is the angle between the vectors $\left(\hat{a}+\hat{b}\right)$ and $\left(\hat{a}+2\hat{b}+\left(\hat{a}+\hat{b}\right)\right)$, then the value of 164 $\cos^2\theta$ is equal to:

(a) $90+27\sqrt{2}$ (b) $45+18\sqrt{2}$

(c) $90+3\sqrt{2}$ (d) $54+90\sqrt{2}$

14. (a)

$$(\hat{a}+\hat{b})\cdot(\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b})), \hat{a}\cdot\hat{b} = \frac{1}{\sqrt{2}}$$

$$= 1+\frac{1}{\sqrt{2}}+\sqrt{2}+2+0+0$$

$$= 3+\frac{3}{\sqrt{2}}$$

$$|\hat{a}+\hat{b}|^2 = 1+1+\sqrt{2} = 2+\sqrt{2}$$

$$\left|\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b})^2\right| = 1+4+4\left(\frac{1}{2}\right)+2\sqrt{2}+0 = 7+2\sqrt{2}$$

$$\sqrt{(2+\sqrt{2})}\sqrt{(7+2\sqrt{2})}\cos\theta = 3+\frac{3}{\sqrt{2}}$$

$$\Rightarrow \quad (2+\sqrt{2})(7+2\sqrt{2})\cos^2\theta = 9+\frac{9}{2}+9\sqrt{2} = \frac{27+18\sqrt{2}}{2}$$

$$\Rightarrow \quad = (14+7\sqrt{2}+4\sqrt{2}+4)$$

$$\Rightarrow \quad \cos^2\theta = \frac{27+18\sqrt{2}}{2}\times\frac{1}{18+11\sqrt{2}} = \frac{(27+18\sqrt{2})(18-11\sqrt{2})}{2(324-242)}$$

$$= \frac{486+324\sqrt{2}-297\sqrt{2}-396}{2\times 82}$$

$$164\cos^2\theta = 90+27\sqrt{2}$$

**Q.15** If $f(\alpha)=\int_1^{\alpha}\frac{\log_{10}t}{1+t}dt, \alpha>0$, then $f(e^3)+f(e^{-3})$ is equal to:

(a) 9 (b) $\frac{9}{2}$

(c) $\frac{9}{\log_e(10)}$ (d) $\frac{9}{2\log_e(10)}$

**15. (d)**

$$f(\alpha) = \int_1^{\alpha}\frac{\log_{10}t}{1+t}dt$$

$$f\left(\frac{1}{\alpha}\right) = \int_1^{\frac{1}{\alpha}}\frac{\log_{10}t}{1+t}dt = \int_1^{\alpha}\frac{-\log_{10}z}{1+\frac{1}{z}}\cdot -\frac{1}{z^2}dz$$

$$= \int_1^{\alpha}\frac{\log_{10}z}{z(z+1)}dz$$

$$f(\alpha)+f\left(\frac{1}{\alpha}\right) = \int_1^{\alpha}\left(\frac{\log_{10}t}{1+t}+\frac{\log_{10}t}{t(1+t)}\right)dt$$

$$= \int_1^{\alpha}\frac{\log_{10}t}{t}dt = \frac{1}{\ell n10}\int_1^{\alpha}\frac{\ell nt}{t}dt = \frac{1}{\ell n10}\left[\frac{(\ell nt)^2}{2}\right]_1^{\alpha} = \frac{(\ell n\alpha)^2}{2\ell n10}$$

$$f(e^3)+f(e^{-3}) = \frac{\left(\ell ne^3\right)^2}{2\ell n10} = \frac{9}{2\ell n10}$$

**Q.16** The area of the region $\left\{(x,y): |x-1| \le y \le \sqrt{5-x^2}\right\}$ is equal to:

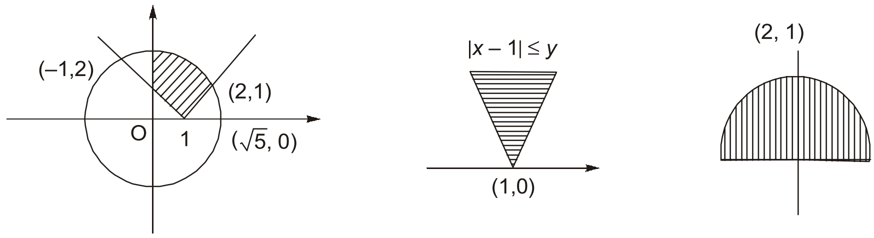
(a) $\frac{5}{2}\sin^{-1}\left(\frac{3}{5}\right)-\frac{1}{2}$ (b) $\frac{5\pi}{4}-\frac{3}{2}$

(c) $\frac{3\pi}{4}+\frac{3}{2}$ (d) $\frac{5\pi}{4}-\frac{1}{2}$

16. (d)

$|x-1| \le y \le \sqrt{5-x^2}$

$$y = |x-1|$$
$$y^2 + x^2 = 5$$
$$y \ge 0 \qquad y = \sqrt{5-x^2}$$




$$(x-1)^2 + x^2 = 5$$
$$\Rightarrow \quad 2x^2 - 2x - 4 = 0$$
$$\Rightarrow \quad x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0 \quad \rightarrow \quad x = -1, 2$$

$$A = \int_{-1}^{2} \left(\sqrt{5-x^2} - |x-1|\right) dx$$

$$= \int_{1}^{2} \sqrt{5-x^2}\, dx - \left(\int_{-1}^{1} (1-x)dx + \int_{1}^{2} (x-1)dx\right)$$

$$\text{Area} = \int_{-1}^{2} \left(\sqrt{5-x^2}\, dx - |x-1|\, dx\right)$$

$$= \int_{-1}^{2} \sqrt{5-x^2}\, dx + \int_{-1}^{1} (x-1)dx \int_{1}^{2} (x-1)dx$$

$$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} + \left[\frac{x^2}{2} - x\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_{1}^{2}$$

$$= \left(1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right) - \left(\frac{-1}{2}(2) + \frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right)\Big]\Big]$$

$$+\left(\frac{1}{2} - 1\right) - \left(\frac{1}{2} + 1\right) - \left\{(2-2) + \left(\frac{1}{2} - 1\right)\right\}$$

$$= \frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right) + 2 - \frac{1}{2} - \frac{3}{2} - \frac{1}{2}$$

$$= \frac{5}{2}\sin^{-1}\left(\frac{2}{\sqrt{5}}\cdot\frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}}\right) - \frac{1}{2}$$

$$= \frac{5}{2}\sin^{-1}(1) - \frac{1}{2} = \frac{5\pi}{4} - \frac{1}{2}$$

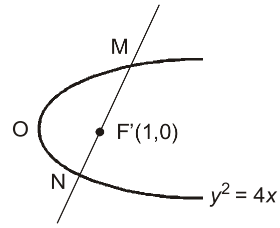
**Q.17** Let the focal chord of the parabola $P: y^2 = 4x$ along the line $L: y = mx + c$, $m > 0$ meet the parabola at the points $M$ and $N$. Let the line $L$ be a tangent to the hyperbola $H: x^2 - y^2 = 4$. If $O$ is the vertex of $P$ and $F$ is the focus of $H$ on the positive $x$-axis, then the area of the quadrilateral $OMFN$ is :

(a) $2\sqrt{6}$ (b) $2\sqrt{14}$

(c) $4\sqrt{6}$ (d) $4\sqrt{14}$

17. (b)

L : $y = mx + c, m > 0$




$\Rightarrow \quad y = m(x-1)$

$$\frac{x^2}{4} - \frac{y^2}{4} = 1$$

$$y = mx \pm \sqrt{4m^2 - 4}$$

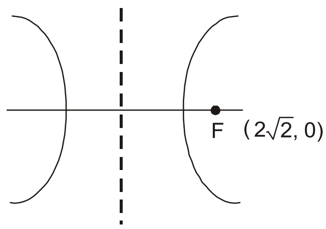
$$\pm\sqrt{4m^2 - 4} = -m, \; m > 0$$

$\Rightarrow \quad \sqrt{4m^2 - 4} = m$

$$4(m^2 - 1) = m^2$$

$\Rightarrow \quad 3m^2 = 4$

$\Rightarrow \quad m = \frac{2}{\sqrt{3}}$




L : $y = \frac{2}{\sqrt{3}}(x-1)$

$$\left.\begin{array}{l} y = \frac{2}{\sqrt{3}}(x-1) \\ y^2 = 4x \end{array}\right\} \Rightarrow \frac{4}{3}(x-1)^2 = 4x$$

$\Rightarrow \quad (x-1)^2 - 3x = 0$

$x^2 - 5x + 1 = 0$

$$\left\{\begin{array}{l} x = \frac{5 \pm \sqrt{21}}{2} \\ y = \frac{2}{\sqrt{3}}\left(\frac{3 \pm \sqrt{21}}{2}\right) = \sqrt{3} \pm \sqrt{7} \end{array}\right.$$

$$M\left(\frac{5+\sqrt{21}}{2}, \sqrt{3}+\sqrt{7}\right), N\left(\frac{5-\sqrt{21}}{2}, \sqrt{3}-\sqrt{7}\right)$$

$$ar(OMFN) = \frac{1}{2}(2\sqrt{2})(\sqrt{3}+\sqrt{7}-(\sqrt{3}-\sqrt{7}))$$

$$= \sqrt{2}(2\sqrt{7}) = 2\sqrt{14}$$

**Q.18** The number of points, where the function f : $R \to R$,
$f(x) = |x-1| \cos|x-2| \sin|x-1| + (x-3)|x^2 - 5x + 4|$, is NOT differentiable, is :

(a) 1 (b) 2

(c) 3 (d) 4

18. (b)

$$f(x) = |x-1| \cos|x-2| \sin|x-1| + (x-3)|x^2-5x+4|$$

$\Rightarrow \quad f(x) = |x-1| \cos(x-2) \sin|x-1| + (x-3)|x-1||x-4|$

$x = 1, 4$ (doubtful points)

Diff. at $x = 1$

$$\lim_{x\to 1} \frac{f(x) - f(4)}{x-1} = \lim_{x \to 1} \frac{|x-1|(\sin|x-1|\cos(x-2) + (x-3)|x-4|)}{x-1}$$

$$\left.\begin{array}{l} \text{RHD} = \lim_{x\to 1^+}(x-3)(4-x) = -6 \\ \text{LHD} = \lim_{x\to 1^-}(x-3)(4-x) = -6 \end{array}\right\} \Rightarrow \text{Not diff. at } x = 1$$

Diff. at $x = 4$

$$\left.\begin{aligned} \text{RHD} &= \lim_{x\to4^+}\frac{3(\sin 3\cos 2)}{0^+} = +\infty \\ \text{LHD} &= \lim_{x\to4^-}\frac{3(\sin 3\cos 2)}{0^-} = -\infty \end{aligned}\right\} \Rightarrow \text{ Not diff. at } x = 4$$

**Q.19** Let $S = \{1, 2, 3,..., 2022\}$. Then the probability, that a randomly chosen number n from the set $S$ such that $HCF(n, 2022) = 1$, is :

(a) $\frac{128}{1011}$ (b) $\frac{166}{1011}$

(c) $\frac{127}{337}$ (d) $\frac{112}{337}$

**19. (d)**

$$S = \{1, 2, 3,...., n, 2022\}$$

$$\text{HCF}(n, 2022) = 1$$

$$2022 = 2 \times 1011 \to 3 \times 337$$

$$2022 = 2\ 5\ 3 \times 337 \quad \text{(Prime factorization)}$$

Let $n(A)$ = no members divisible by 2 = 1011

Let $n(B)$ = no members divisible by 3 = 674

Let $n(C)$ = no members divisible by 337 = 6

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

$$= 1011 + 674 + 6 - 337 - 2 - 3 + 1$$

$$= 1691 - 342 + 1$$

$$= 1692 - 342 = 1350$$

$$n(A \cap B) = 337$$

$$n(B \cap C) = 2$$

$$n(C \cap A) = 3$$

$$n((A \cup B \cup C)') = 2022 - 1350 = 672$$

$$\text{Prob.} = \frac{672}{2022} = \frac{336}{1011} = \frac{112}{337}$$

**Q.20** Let $f(x) = 3^{(x^2-2)^3+4}$, $x \in R$. Then which of the following statements are true?

$P$ : $x = 0$ is a point of local minima of $f$

$Q$ : $x = \sqrt{2}$ is a point of inflection of $f$

$R$ : $f'$ is increasing for $x > \sqrt{2}$

(a) Only $P$ and $Q$ (b) Only $P$ and $R$

(c) Only $Q$ and $R$ (d) All $P$, $Q$ and $R$

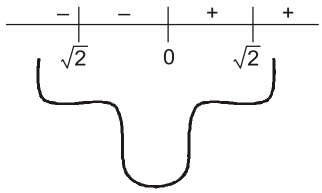
**20. (d)**

$$f(x) = 3^{(x^2-2)^3+4} = 81.3^{(x^2-2)^3}$$

$$f'(x) = 81.3^{(x^2-2)^3}.\ell n3.3(x^2-2)^2.2x$$

$$f'(x) = 486\,\ell n3.x(x^2-2)^3.3^{(x^2-2)^3}$$

From graph : P, Q, R



# SECTION - B

### (Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

**Q.1** Let $S = \{\theta \in (0, 2\pi) : 7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2\}$. Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)\,x + 6\sin^2\theta = 0,\ \theta \in S$, is .....

**1.** **(16)**

$$\frac{7}{2}(1+\cos 2\theta) - \frac{3}{2}(1-\cos 2\theta) - 2\cos^2 2\theta = 2$$

Put $\cos 2\theta = t$

Equation : $2t^2 - 5t = 0$

$$t(2t-5) = 0$$

$$t = 0, \frac{5}{2}$$

$$\cos 2\theta = 0,\ 0 < 2\theta < 4\pi$$

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$x_1 + x_2 = 2(\tan^2\theta + \cot^2\theta) \quad \Rightarrow \quad ?$$

$$= 2(1+1) + 2(1+1) + 2(1+1) + 2(1+1) = 16$$

**Q.2** Let the mean and the variance of 20 observations $x_1, x_2, \ldots, x_{20}$ be 15 and 9, respectively. For $\alpha \in R$, if the mean of $(x_1+\alpha)^2, (x_2+\alpha)^2, \ldots, (x_{20}+\alpha)^2$ is 178, then the square of the maximum value of $\alpha$ is equal to .....

**2.** **(4)**

$$\bar{x} = 15$$

$$\Rightarrow \sum_{i=1}^{20} x_i = 300$$

$$\frac{\sum_{i=1}^{20}(x_i - 15)^2}{20} = 9$$

$$\sum_{i=1}^{20}(x_i - 15)^2 = 180$$

$$\Rightarrow \sum x_i 2 - 30\underbrace{\sum x_i}_{9000} + \underbrace{\sum 225}_{4500} = 180$$

$$\sum x_i 2 = 4680$$

$$\sum_{i=1}^{20}(x_i + \alpha)^2) = 178 \times 20 = 3560$$

$$4680 + 2\alpha(300) + 20\alpha^2 = 3560$$

$$\alpha^2 + 30\alpha + 234 - 178 = 0$$

$$\Rightarrow \alpha^2 + 30\alpha + 56 = 0$$

$$\Rightarrow (\alpha + 2)(\alpha + 28) = 0$$

$\Rightarrow \quad \alpha = -2, \ -28$

$\alpha_{max}2 = (-2)^2 = 4$

**Q.3** Let a line with direction ratios $a, -4a, -7$ be perpendicular to the lines with direction ratios $3, -1, 2b$ and $b, a, -2$ . If the point of intersection of the line $\frac{x+1}{a^2+b^2} = \frac{y-2}{a^2-b^2} = \frac{z}{1}$ and the plane $x - y + z = 0$ is $(\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma$ is equal to ....

**3.** **(10)**

$$(a, -4a, -7) \cdot (3, -1, 2b) = 0$$

$$\Rightarrow 3a + 4a - 14b = 0$$

$$\Rightarrow a - 2b = 0 \quad ...(i)$$

$$(a, -4a, -7) \cdot (b, a, -2) = 0$$

$$\Rightarrow ab - 4a^2 + 14 = 0 \quad ...(ii)$$

(*i*) and (*ii*)

$$\Rightarrow 2b^2 - 16b^2 + 14 = 0$$

$$\Rightarrow b^2 = 1$$

$$\Rightarrow a = 2b = \pm 2$$

Line : $\frac{x+1}{5} = \frac{y-2}{3} = \frac{z}{1} = t$

P $(-1 + 5t, 2 + 3t, t)$

Plane : $x - y + z = 0$

$$-1 + 5t - 2 - 3t + t = 0$$

$$3t - 3 = 0$$

$$\Rightarrow t = 1$$

$P(4, 5, 1)$

$$4 + 5 + 1 = 10$$

**Q.4** let $a_1, a_2, a_3,$ ...... be an A.P. If $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to ........

**4.** **(16)**

$$\sum_{r=1}^{\infty} \frac{a+(r-1)d}{2^r} = 4$$

$$4(a + d) = ?$$

$$\Rightarrow a\sum_{r=1}^{\infty}\left(\frac{1}{2}\right)^r + d\sum_{r=1}^{\infty}(r-1)\left(\frac{1}{2}\right)^r = 4$$

$$\sum_{r=1}^{\infty}\left(\frac{1}{2}\right)^r = \frac{1/2}{1-\frac{1}{2}} = 1$$

$$\sum_{r=0}^{\infty} x^r = \frac{1}{1-x}$$

$$\sum_{r=0}^{\infty} r x^{r-1} = \frac{1}{(1-x)^2}$$

$$= \sum_{r=1}^{\infty}(r-1)\left(\frac{1}{2}\right)^r$$

$$= \sum_{k=0}^{\infty} k\left(\frac{1}{2}\right)^{k+1}$$

$$= \left(\frac{1}{2}\right)^2 \sum_{k=0}^{\infty} k\left(\frac{1}{2}\right)^{k-1}$$

$$= \frac{1}{4}\cdot\frac{1}{\left(1-\frac{1}{2}\right)^2} = 1$$

$$a + d = 4$$

$$\Rightarrow \quad 4(a+d) = 16$$

**Q.5** Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ in the increasing powers of $\frac{1}{\sqrt[4]{3}}$ be $\sqrt[4]{6}:1$. If the sixth term from the beginning is $\frac{\alpha}{\sqrt[4]{3}}$, then $\alpha$ is equal to ........

**5.** **(84)**

$$T_5 = {}^nC_4\left(2^{\frac{1}{4}}\right)^{n-4}\cdot\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}\right)^4 = {}^nC_4\cdot 2^{\frac{n-4}{4}}\cdot\frac{1}{3}$$

$$T_{n-3} = {}^nC_{n-4}\left(2^{\frac{1}{4}}\right)^4\cdot\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}\right)^{n-4} = {}^nC_{n-4}\cdot 2\cdot\left(\frac{1}{3}\right)^{\frac{n-4}{4}}$$

$$\frac{T_5}{T_{n-3}} = \frac{2^{(n-8)/4}}{\left(\frac{1}{3}\right)^{(n-3)/4}} = 6^{(n-8)/4} = 6^{\frac{1}{4}}$$

$$\Rightarrow \quad n = 9$$

$$\Rightarrow \quad \alpha = 84 = \frac{84}{3^{\frac{1}{4}}}n$$

$$T_6 = {}^9C_5\left(2^{\frac{1}{4}}\right)^4\cdot\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}\right)^5$$

$$= {}^9C_5\cdot 2\cdot\frac{1}{3^{5/4}} = \frac{2}{3}\cdot\frac{1}{3^{\frac{1}{4}}}\cdot\frac{9\times8\times7\times6}{4\times3\times2\times1}$$

**Q.6** The number of matrices of order 3 × 3, whose entries are either 0 or 1 and the sum of all the entries is a prime number is .....

**6.** **(282)**

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}, a_2, b_2, c_2 \in \{0,1\}$$

$S = a_1 + a_2 + a_3 + b_1 + b_2 + b_3 + c_1 + c_2 + c_3$ is prime.

$0 \le s \le 9$

Prime value = 2, 3, 5, 7

For $S = 2,\ 1 + 1 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \rightarrow \frac{9!}{2!7!} = 36$

$S = 3, \rightarrow \frac{9!}{3!6!} = \frac{9\times8\times7}{24} = 84$

$S = 5, \rightarrow \frac{9!}{5!4!} = \frac{9\times8\times7\times6}{24} = 126$

$S = 7 \rightarrow \frac{9!}{7!2!} = 36$

Total number of matrices = 36 + 84 + 126 + 36 = 282

**Q.7** Let $p$ and $p + 2$ be prime numbers and let

$$\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$$

Then the sum of the maximum values of $\alpha$ and $\beta$, such that $p^{\alpha}$ and $(p + 2)^{\beta}$ divide $\Delta$, is ......

**7.** **(4)**

$\Delta = p!(p + 1)!\,(p + 2)!\ \Delta_1 = 2p!(p + 1);\ (p + 2)! = 2(p!)^3 \,.\, (p + 1)^2 \,.\, (p + 2)^1 \rightarrow \alpha + \beta = 3 + 1 = 4$

$$\Delta_1 = \begin{vmatrix} 1 & p+1 & (p+2)(p+1) \\ 1 & p+2 & (p+3)(p+2) \\ 1 & p+3 & (p+4)(p+3) \end{vmatrix} = \begin{vmatrix} 0 & -1 & (p+2)(-2) \\ 0 & -1 & (p+3)(-2) \\ 1 & p+3 & (p+4)(p+3) \end{vmatrix}$$

$= 2\,(p + 3) - 2(p + 2) = 2$

**Q.8** If $\frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + \frac{1}{4\times5\times6} + \ldots + \frac{1}{100\times101\times102} = \frac{k}{101}$, then 34 $k$ is equal to .......

**8.** **(286)**

$$t_n = \frac{1}{(n+1)(n+2)(n+3)},\quad n = 1, 2, 3, ......, 99$$

$$= \frac{1}{2}\left(\frac{1}{(n+1)(n+2)} - \frac{1}{(n+2)(n+3)}\right) = V_n - V_{n+1}$$

$$\sum_{n=1}^{99} t_n = t_1 + t_2 + t_3 + .... + t_{99}$$

$$= V_1 - V_2 + V_2 - V_3 + V_{99} - V_{100}$$

$$= V_1 - V_{100}$$

$$= \frac{1}{2}.\frac{1}{6} - \frac{1}{2}.\frac{1}{101.102}$$

$$= \frac{1}{12}\left(1 - \frac{1}{101\times17}\right)$$

$$= \frac{1}{12}\left(\frac{1716}{101\times17}\right) = \frac{143}{101\times17} = \frac{k}{101}$$

$\Rightarrow$ $k = \frac{143}{17}$

$34k = 286$

**Q.9** Let $S = \{4, 6, 9\}$ and $T = \{9, 10, 11, \ldots, 1000\}$.
If $A = \{a_1 + a_2 + \ldots + a_k : k \in N, a_1, a_2, a_3, \ldots, a_k \in S\}$, then the sum of all the elements in the set $T - A$ is equal to ........

**9.** **(11)**

$$T = \{9, 10, 11, 12, \ldots, 1000\}$$
$$S = \{4, 6, 9\}$$
$$A = \{a_1 + a_2 + \ldots + a_k : k \in N, a_i \in S\}$$

Let 4 appear x no. of times
6 appear y no. of times
9 appear z no. of times
10 then set A has $n$ element $4x + 6y + 9z$

Concept : Let $a$ and $b$ are co-prime numbers, then members from $(a - 1)(b - 1)$ and more can be expressed in the form $ax + by$ where $x, y, \in \{0, 1, 2, \ldots\}$

So, all the numbers of the form

$2y + 3z$ are $(2 - 1) . (3 - 1), \ldots$

i.e. 6, 7, 8, 9, 10, 11,....

form : $2 + t$, $t = 0, 1, 2, 3, \ldots$

So, $6y + 9z = 3(2y + 3z) = 3(2 + t) = 6 + 3t$, $t = 0, 1, 2, 3, \ldots$

$\Rightarrow 4x + 6y + 9z$

$= 4x + 6 + 3t$

$= 4x + 3t + 6$

Now, all the numbers of the form $4x + 3t$ start from $(4 - 1)(3 - 1) = 6$ and 6, 7, 8, 9, ..... $\rightarrow$ form:

$6 + k$

$k = 0, 1, 2, 3, \ldots$

So, $4x + 6y + 9z$

$= 4x + 3t + 6$

$= 6 + k + 6$

$= 12 + k (k = 0, 1, 2, \ldots)$

$\Rightarrow$ Numbers of the form $4x + 6y + 9z$ are 12, 13, 14, 15.....

But $9 \le 4x + 6y + 9z \le 1000$ and $9 = 4(0) + 6(0) + 9(1)$ & $10 = 4(1) + 6(1) + 9(0)$

But 11 can't be written in form $4x + 6y + 9z$ .

$\Rightarrow A = \{9, 10, 12, 13, 14, 15, \ldots, 1000\}$

$T = \{9, 10, 11, 12, 13, 14, 15, \ldots, 1000\}$

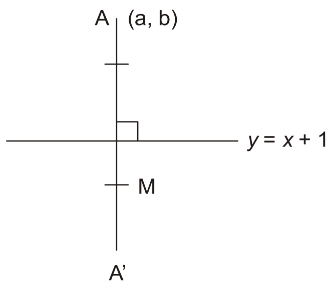
$T - A = \{11\}$

$\Rightarrow$ sum of element of $T - A$ is 11.

**Q.10** Let the mirror image of a circle $c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$ in line $y = x + 1$ be $c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$. If $r$ is the radius of circle $c_2$, then $\alpha + 6r^2$ is equal to ........

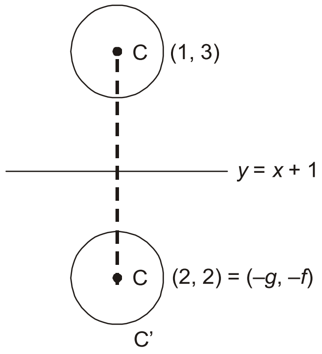
**10.** **(12)**

$$(x - 1)^3 + (y - 3)^2 = 10 - \alpha$$



$$\left.\begin{array}{l} x - y = -1 \\ x + y = a + b \end{array}\right\} \rightarrow m\left(\frac{a+b-1}{2}, \frac{a+b+1}{2}\right)$$

$$A' = 2m - A = (b - 1, a + 1)$$




$$C_2 : x^2 + y^2 + 2gx + 2fy + \frac{38}{5} = 0$$

$$r = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{4 + 4 - \frac{38}{5}} = \sqrt{\frac{2}{5}}$$

$$C_1 : \sqrt{\frac{2}{5}} = \sqrt{1 + 9 - \alpha} = \sqrt{10 - \alpha}$$

$$10 - \alpha = \frac{2}{5}$$

$\Rightarrow$ $$\alpha = 10 - \frac{2}{5} = \frac{48}{5}$$

$$\alpha + 6r^2 = \frac{48}{5} + 6\left(\frac{2}{5}\right) = \frac{60}{5} = 12$$

❍❍❍❍