MADE EASY & NEXT IAS GROUP

PRESENT

MENNIT

NEET | IIT-JEE | FOUNDATION

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JEE (MAIN) 2022

Test Date: 26th July 2022 (First Shift)

PAPER-1

Questions with Solutions

Time : 3 Hours **Maximum Marks: 300**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

IMPORTANT INSTRUCTIONS:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts**. **Part-A** is *Physics*, **Part-B** is *Chemistry* and **Part-C** is *Mathematics*. Each part has only two sections: **Section-A** and **Section-B**.
4. **Section – A :** Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** contains 20 multiple choice questions which have only one correct answer. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (1 – 10)** contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

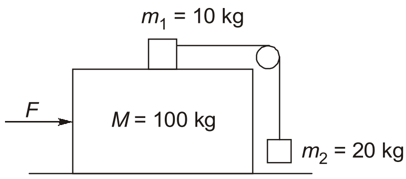
PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

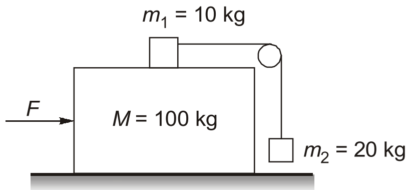
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q.1 Three masses $M = 100$ kg, $m_1 = 10$ kg and $m_2 = 20$ kg are arranged in a system as shown in figure. All the surface are frictionless and strings are inextensible and weightless. The pulleys are also weightless and frictionless. A force F is applied on the system so that the mass m_2 moves upward with an acceleration of 2 ms^{-2}. The value of F is : (Take g = 10 ms^{-2})



(a) 3360 N (b) 3380 N
(c) 3120 N (d) 3240 N

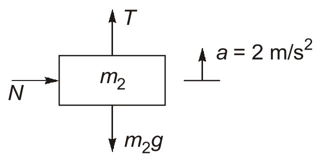
1. **(a)**



Given Acceleration of m_2 is 2 m / s^2 (upward)

$N = 20a$

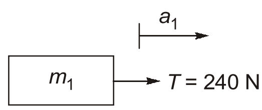
$$\begin{aligned} T - m_2 g &= m_2 a \\ T - 20g &= 20a \\ T &= 200 + 20 \times 2 \\ T &= 240 \text{ N} \\ T &= m_1 a_1 \\ 240 &= 10 a_1 \\ a_1 &= 24 \text{ m/s}^2 \end{aligned}$$

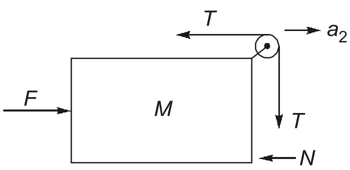


$$\Rightarrow \quad Ta + Ta_1 - Ta_2 = 0$$

$$\Rightarrow \quad a + a_1 = a_2$$

$$\begin{aligned} a_2 &= 24 + 2 = 26 \text{ m/s}^2 \\ F - T - N &= Ma_2 \\ F - 240 &= 100 \times 26 + 20 \times 26 \\ F &= 2600 + 240 + 20 \times 26 \\ F &= 3360 \text{ N} \end{aligned}$$





Q.2 A radio can tune to any station in 6 MHz to 10 MHz band. The value of corresponding wavelength bandwidth will be :

(a) 4 m (b) 20 m
(c) 30 m (d) 50 m

2. (b)

$$\lambda_1 = 6 \text{ MHz} = 6 \times 10^6 \text{ Hz}$$

$$\lambda_1 = \frac{C}{\nu_1} = \frac{3\times10^8}{6\times10^6} = 50 \text{ m}$$

$$\lambda_2 = 10 \text{ MHz} = 10 \times 10^6 \text{ Hz}$$

$$\lambda_2 = \frac{C}{\nu_2} = 10\times10^6 = \frac{3\times10^8}{10\times10^6} = 30 \text{ m}$$

$\Rightarrow$ Wavelength bandwidth

$$= |\lambda_1 - \lambda_2| = 20 \text{ m}$$

Q.3 The disintegration rate of a certain radioactive sample at any instant is 4250 disintegrations per minutes later, the rate becomes 2250 disintegrations per minute. The approximate decay constant is :
(Take $\log_{10} 1.88 = 0.274$)

(a) 0.02 min^{-1}
(b) 2.7 min^{-1}
(c) 0.063 min^{-1}
(d) 6.3 min^{-1}

3. (c)

$$A = Aoe^{-\lambda t}$$

$$\Rightarrow \quad 2250 = 4250e^{-\lambda t}$$

$$\Rightarrow \quad e^{-\lambda t} = \frac{2250}{4250}$$

$$\Rightarrow \quad -\lambda t \log_{10}(e) = -\log_{10}(1.88)$$

$$\Rightarrow \quad \lambda \times 10 \times 0.434 = 0.274$$

$$\Rightarrow \quad \lambda = \frac{0.274}{10\times0.434} = 0.63 \text{ min}^{-1}$$

Q.4 A parallel beam of light of wavelength 900 nm and intensity 100 Wm^{-2} is incident on a surface perpendicular to the beam. The number of photons crossing 1 cm^2 area perpendicular to the beam in one second is :

(a) 3×10^{16}
(b) 4.5×10^{16}
(c) 4.5×10^{17}
(d) 4.5×10^{20}

4. (b)

$$E = IA$$

$$= 100 \times 1 \times 10^{-4}$$

$$= 10^{-2} \text{ W}$$

$$E = \dot{n}h\frac{c}{\lambda}$$

$$10^{-2} = \dot{n} \times \frac{6.64\times10^{-34}\times3\times10^8}{900\times10^{-9}}$$

$$\dot{n} = \frac{10^{-2}\times9\times10^{-7}}{6.64\times10^{-34}\times3\times10^8}$$

$$\dot{n} = \frac{9}{3\times6.64}\times10^{17}$$

$$\dot{n} = 4.5 \times 10^{16}$$

Q.5 In Young's double slit experiment, the fringe width is 12 mm. If the entire arrangement is placed in water of refractive index $\frac{4}{3}$, then the fringe width becomes (in mm) :

(a) 16 (b) 9
(c) 48 (d) 12

5. (b)

$$\beta = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$$

$$\beta' = \frac{\beta}{\mu} = \frac{12 \times 10^{-3}}{4/3} = 9 \times 10^{-3} \text{ m}$$

$$\beta' = 9 \text{ mm}$$

Q.6 The magnetic field of a plane electromagnetic wave is given by :

$\vec{B} = 2 \times 10^{-8} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11})\hat{j}T$

The amplitude of the electric field would be :

(a) 6 Vm^{-1} along x-axis (b) 3 Vm^{-1} along z-axis
(c) 6 Vm^{-1} along z-axis (d) 2×10^{-8} Vm^{-1} along z-axis

6. (c)

$$\text{Speed of light, } C = \frac{w}{k} = \frac{1.5 \times 10^{11}}{0.5 \times 10^3} = 3 \times 10^8 \text{ m/s}$$

So,
$$E_0 = B_0 C$$
$$= 2 \times 10^{-8} \times 3 \times 10^8$$
$$= 6 \text{ V/m}$$

Direction will be along z-axis.

Q.7 In a series LR circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2. The ratio $\frac{P_1}{P_2}$ is:

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) 2 : 1

7. (b)

$$P_1 = \cos\phi = \frac{R}{z} = \frac{R}{\sqrt{X_L^2 + R^2}}$$

$$P_1 = \cos\phi = \frac{R}{R\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$P_1 = \frac{1}{\sqrt{2}}$$

$$P_2 = \cos\phi = 1$$

So,
$$\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$$

Q.8 A charge particle is moving in uniform magnetic field $(2\hat{i} + 3\hat{j})T$. If it has an acceleration of $(\alpha\hat{i} - 4\hat{j})$ m/s^2, then the value of α will be :

(a) 3 (b) 6
(c) 12 (d) 2

8. (b)

$$\vec{a} \perp \vec{B}$$
$$\Rightarrow \quad \vec{a}\cdot\vec{B} = 0$$
$$\Rightarrow \quad (2\alpha - 12) = 0$$
$$\alpha = 6$$

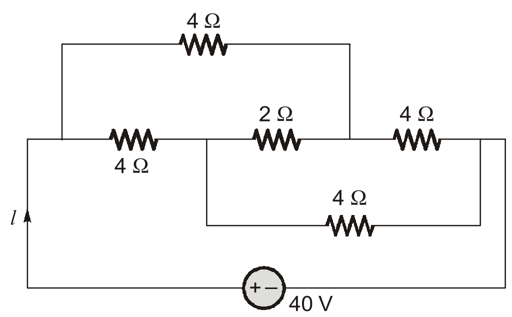
Q.9 B_X and B_Y are the magnetic fields at the centre of two coils X and Y respectively each carrying equal current. If coil X has 200 turns and 20 cm radius and coil Y has 400 turns and 20 cm radius, the ratio of B_X and B_Y is:

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 4 : 1

9. (b)

$$B = N\frac{\mu_0 I}{2\mu}$$
$$B \;\alpha\; \frac{N}{\mu}$$
$$\frac{\beta_x}{\beta_y} = \frac{N_x}{r_x} \times \frac{r_y}{N_y}$$
$$\frac{B_x}{B_y} = \frac{200}{20} \times \frac{20}{400} = \frac{1}{2}$$

Q.10 The current I in the given circuit will be :



(a) 10 A (b) 20 A
(c) 4 A (d) 40 A

10. (a)

$$R = \frac{8}{2}\Omega = 4\,\Omega \text{ (What stone bridge)}$$
$$I = \frac{V}{R} = \frac{40}{4} = 10\text{ A}$$

Q.11 The total charge on the system of capacitors $C_1 = 1\,\mu F$, $C_2 = 2\,\mu F$, $C_3 = 4\,\mu F$ and $C_4 = 3\,\mu F$ connected in parallel is :
(Assume a battery of 20 V is connected to the combination)

(a) 200 μC (b) 200 C
(c) 10 μC (d) 10 C

11. (a)

$$C_{eq} = C_1 + C_2 + C_3 + C_4$$
$$= 1 + 2 + 4 + 3$$
$$= 10\,\mu F$$
$$Q = Ce_q V$$
$$= (10 \times 20)\,\mu C$$
$$Q = 200\,\mu C$$

Q.12 When a particle executes Simple Hormonic Motion, the nature of graph of velocity as a function of displacement will be :

(a) Circular (b) Elliptical

(c) Sinusoidal (d) Straight line

12. (b)

$$x = A \sin \omega t$$
$$\Rightarrow \quad V = A\omega \cos \omega t$$
$$\Rightarrow \quad V = \pm\omega\sqrt{A^2 - x^2}$$
$$\frac{V^2}{\omega^2} + x^2 = A^2$$

Elliptical

Q.13 7 mol of a certain monoatomic ideal gas undergoes a temperature increase of 40 K at constant pressure. The increase in the internal energy of the gas in this process is :

(Given $R = 8.3\ JK^{-1}\ mol^{-1}$)

(a) 5810 J (b) 3486 J

(c) 11620 J (d) 6972 J

13. (b)

$$\Delta u = nC_V\Delta T$$
$$= n\frac{3R}{2}\Delta T$$
$$= 7 \times \frac{3}{2} \times 8.3 \times 40$$

Constant Entropy means Adiabatic process

$$= 3486\ J$$

Q.14 A monoatomic gas at pressure P and volume V is suddenly compressed to one eighth of its original volume. The final pressure at constant entropy will be :

(a) P (b) 8 P

(c) 32 P (d) 64 P

14. (c)

Constant Entropy means Adiabatic process

$$Pv^y = C$$
$$P_1v_1^y = P_2v_2^y$$
$$P_2 = P_1\left(\frac{v_1}{v_2}\right)^y = P\left(\frac{v}{\frac{1}{8}v}\right)^y$$
$$P_2 = P(8)\frac{5}{3}$$
$$P_2 = 32P$$

Q.15 A water drop of radius 1 cm is broken into 729 equal droplets. If surface tension of water is 75 dyne /cm, then the gain in surface energy upto first decimal place will be :
(Given $\pi = 3.14$)

(a) 8.5×10^{-4} J
(b) 8.2×10^{-4} J
(c) 7.5×10^{-4} J
(d) 5.3×10^{-4} J

15. (c)

$$\frac{4}{3}\pi R^3 = 729\frac{4}{3}\pi r^3$$
$$R^3 = 729r^3$$
$$R = (729)\frac{1}{3}(r)\frac{1}{3}$$
$$R = 9r$$
$$\Delta u = T(4\pi r^2)\times 729 - T\times 4\pi R^2$$
$$= T\times 4\pi\left(729\left(\frac{R}{9}\right)^2 - R^2\right)$$
$$= T\times 4\pi\left(\frac{729R^2}{81} - R^2\right)$$
$$= T\times 4\pi\times 8R^2$$
$$= 75\times 10^{-5}\times 4\pi\times\frac{8\times 1}{100}$$
$$= 75.39\times 10^{-5}\text{ J}$$
$$\Delta u = 7.5\times 10^{-4}\text{ J}$$

Q.16 The percentage decrease in the weight of a rocket, when taken to a height of 32 km above the surface of earth will, be :
(Radius of earth = 6400 km)

(a) 1%
(b) 3%
(c) 4%
(d) 0.5%

16. (a)

$$g^1 = g\left(1-\frac{2h}{R}\right) = g\left(1-2\times\frac{32}{6400}\right)$$
$$g^1 = \frac{99g}{100} = 0.99g$$
$$\%\text{ decrease is wt } = \frac{g-g^1}{g}\times 100 = 1\%$$

Q.17 As per the given figure, two blocks each of mass 250 g are connected to a spring of spring constant 2 Nm^{-1}. If both are given velocity v in opposite directions, then maximum elongation of the spring is :

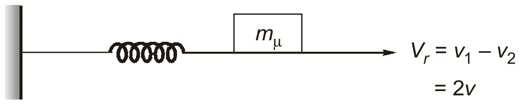
(a) $\frac{v}{2\sqrt{2}}$
(b) $\frac{v}{2}$
(c) $\frac{v}{4}$
(d) $\frac{v}{\sqrt{2}}$

17. (b)

Let $m = 250$ g = 0.25 kg

By reduced mass method

$$m_r = \frac{m_1 m_2}{m_1 + m_2} = \frac{mm}{m+m} = \frac{m}{2}$$



By wet

$$W_{SP} = \Delta K.E$$

$$\Rightarrow \quad -\frac{1}{2}kx^2 = 0 - \frac{1}{2}\left(\frac{m}{2}\right)(2v)^2$$

$$\frac{1}{2}kx^2 = \frac{1}{2} \times \frac{m}{2} \times 4v^2$$

$$\frac{kx^2}{2} = mv^2$$

$$\frac{2}{2}x^2 = 0.25v^2$$

$$x^2 = 0.25v^2$$

$$x = 0.5v$$

$$x = \frac{v}{2}$$

Q.18 A monkey of mass 50 kg climbs on a rope which can withstand the tension (T) of 350 N. If monkey initially climbs down with an acceleration of 4 m/s^2 and then climbs up with an acceleration of 5 m/ s^2. Choose the correct option (g = 10 m/s^2).

(a) T = 700 N while climbing upward

(b) T = 350 N while climbing downward

(c) Rope will break while climbing upward

(d) Rope will break while going downward

18. (c)

For climbing downward

$$50g - T = 50a$$

$$T = 500 - 50 \times 4$$

$$= 300 \text{ N} < 350 \text{ N}$$

for climbing upwards

$$T - 50g = 50a$$

$$T - 500 = 50 \times 5$$

$$T = 750 \text{ N} > 350 \text{ N}$$

Q.19 Two projectiles thrown at 30° and 45° with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is :

(a) $1 : \sqrt{2}$ (b) 2 : 1

(c) $\sqrt{2} : 1$ (d) 1 : 2

19. (c)

$$H_1 = H_2$$

$$\frac{u_1^2 \sin^{25}\theta_1}{2g} = \frac{u_2^2 \sin^2\theta_2}{2g}$$

$$\Rightarrow \quad u_1^2(\sin 30°)^2 = u_2^2(\sin 45°)^2$$

$$\Rightarrow \quad \frac{u_1^2}{4} = \frac{u_2^2}{2}$$

$$\Rightarrow \quad u_1^2 = 2u_2^2$$

$$\Rightarrow \quad \frac{u_1}{u_2} = \sqrt{2}$$

Q.20 A screw gauge of pitch 0.5 mm is used to measure the diameter of uniform wire of length 6.8 cm, the main scale reading is 1.5 mm and circular scale reading is 7. The calculated curved surface area of wire to appropriate significant figures is :
[Screw gauge has 50 divisions on its circular scale]

(a) 6.8 cm^2 (b) 3.4 cm^2
(c) 3.9 cm^2 (d) 2.4 cm^2

20. (b)

$$\text{Least count} = \frac{0.5}{50}\text{mm} = 0.01\,\text{mm}$$

$$\text{Diameter, } d = 1.5 + 7 \times 0.01$$

$$\therefore \quad \text{Surface Area} = (2\pi r)l$$

$$= \pi d l$$

$$= 3.142 \times \frac{1.57}{10} \times 6.8$$

$$= 3.354\ \text{cm}^2$$

$$= 3.4\ \text{cm}^2$$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

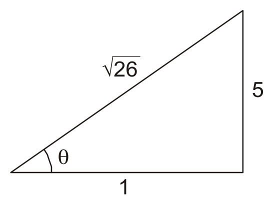
Q.1 If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1 - x)$. The y component vector of the initial velocity is ______ $\hat{j}$.
(Take $g = 10$ m/s^2)

1. (5)

$$y = x5(1 - x) = x\tan\theta\left(1 - \frac{x}{R}\right)$$

$$\tan\theta = 5,\ R = 1$$

$$\sin\theta = \frac{5}{\sqrt{26}},\ \cos\theta = \frac{1}{\sqrt{26}}$$



$$R = \frac{u^2 \sin 2\theta}{g} = 1$$

$$\Rightarrow \quad \frac{u^2}{10} \times 2\sin\theta\cos\theta = 1$$

$$\Rightarrow \quad \frac{u^2}{10} \times 2 \times \frac{5}{\sqrt{26}} \times \frac{1}{\sqrt{26}} = 1$$

$$u^2 = 26 \quad \Rightarrow \quad u = \sqrt{26} \text{ m/s}$$

y-component of initial velocity

$$= u \sin\theta$$

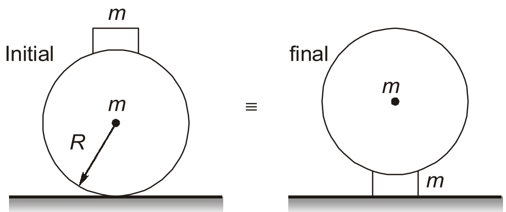
$$= \sqrt{26} \times \frac{5}{\sqrt{26}}$$

$$= 5 \text{ m/s}$$

Q.2 A disc of mass 1 kg and radius R is free to rotate about a horizontal axis passing through its centre and perpendicular to the plane of disc. A body of same mass as that of disc is fixed at the highest point of the disc. Now the system is released, when the body comes to the lowest position, its angular speed will be $4\sqrt{\frac{x}{3R}}$ rad s^{-1} where x = ________. (g = 10 ms^{-2})

2. **(5)**

Loss in P.E = Gain in K.E



$$2\, mg\, R = \frac{1}{2}\left(\frac{1}{2}mR^2 + mR^2\right)\omega^2$$

$$\omega = \sqrt{\frac{8g}{3R}} = 4\sqrt{\frac{g}{2 \times 3R}}$$

$$x = \frac{g}{2} = 5$$

Q.3 In an experiment to determine the Young's modulus of wire of a length exactly 1 m, the extension in the length of the wire is measured as 0.4 mm with an uncertainty of ±0.02 mm when a load of 1 kg is applied. The diameter of the wire is measured as 0.4 mm with an uncertainty of ±0.01 mm.
The error in the measurement of Young's modulus (ΔY) is found to be $x \times 10^{10}$ Nm^{-2}. The value of x is ________.
(take g = 10 ms^{-2})

3. **(1.99)**

$$L = 1 \text{ m}$$

$$\Delta L = 0.4 \times 10^{-3} \text{ m}$$

$$m = 1 \text{ kg}$$

$$d = 0.4 \times 10^{-3} \text{ m}$$

$$\frac{F}{A} = y\frac{\Delta L}{L}$$

$$y = \frac{FL}{A\Delta L} = \frac{(mg)\times 1}{\left(\frac{\pi d^2}{4}\right)\times 0.4\times 10^{-3}}$$

$$= \frac{10\times 4}{\pi(0.4\times 10^{-3})\times 0.4\times 10^{-3}}$$

$$y = \frac{40}{\pi(0.4\times 10^{-3})^3}$$

$$y = \frac{40\times 7}{22\times 64\times 10^{-3}\times 10^{-9}}$$

$$y = 0.199\times 10^{12}\ \text{N/m}^2$$

$$\frac{\Delta y}{y} = \frac{\Delta F}{F}+\frac{\Delta L}{L}+\frac{\Delta A}{A}+\Delta\left(\frac{\Delta L}{\Delta L}\right)$$

$$= \frac{2\Delta d}{d}+\Delta\left(\frac{\Delta L}{\Delta L}\right)$$

$$= 2\times\frac{0.01}{0.4}+\frac{0.02}{0.4}$$

$$\Delta y = 0.1\times 0.199\times 10^{12} = 1.99\times 10^{10}$$

$$= 1.99$$

Q.4 When a car is approaching the observer, the frequency of horn is 100 Hz. After passing the observer, it is 50 Hz. If the observer moves with the car, the frequency will be $\frac{x}{3}$ Hz where x = _______.

4. (200)

$$f_1 = 100 = f_0\left(\frac{C}{C-V_s}\right)$$

C = Speed of sound

V_s = Speed of source

$$f_2 = 50 = f_0 = \left(\frac{C}{C+V_s}\right)$$

$$\frac{f_1}{f_2} = 2 = \frac{C+V_s}{C-V_s}$$

$$\Rightarrow \quad 2C-2V_s = C+V_s$$

$$3V_s = C$$

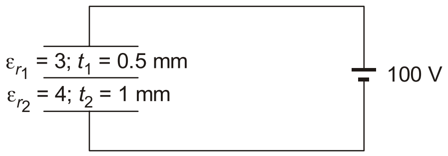
$$V_s = \frac{C}{3}$$

$$100 = f_0\frac{C}{\frac{2C}{3}} = \frac{3}{2}f_0$$

$$f_0 = \frac{200}{3} = \frac{x}{3}$$

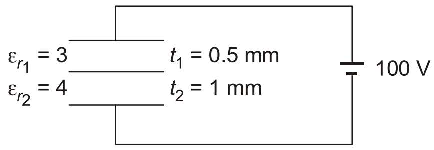
$$x = 200$$

Q.5 A composite parallel plate capacitor is made up of two different dielectric materials with different thickness (t_1 and t_2) as shown in figure. The two different dielectric materials are separated by a conducting foil F. The voltage of the conducting foil is _________ V.



5. **(60)**

Capacitance of each capacitor



$$C_1 = \frac{A3\varepsilon_0}{\frac{1}{2}} = 6A\varepsilon_0$$

$$C_2 = A4\varepsilon_0 = 4A\varepsilon_0$$

Equivalent capacitance

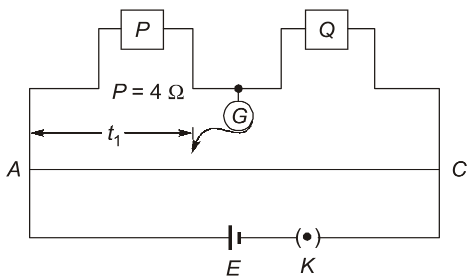
$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{24}{10}A\varepsilon_0$$

$$q_{Net} = C_{eq}(\Delta V) = 240A\varepsilon_0$$

$$\Delta V_2 = \frac{240A\varepsilon_0}{4A\varepsilon_0} = 60 \text{ V}$$

$$V_{foil} = 60 \text{ V}$$

Q.6 Resistances are connected in a meter bridge circuit as shown in the figure. The balancing length l_2 is 40 cm. Now an unknown resistance x is connected in series with P and new balancing length is found to be 80 cm measured from the same end. Then the value of x will be ________ Ω.



6. **(20)**

Initially, $$\frac{P}{Q} = \frac{40}{60} = \frac{2}{3} \quad ...(1)$$

Finally, $$\frac{P+x}{Q} = \frac{80}{20} = \frac{4}{1} \quad ...(2)$$

(2) ÷ (1)

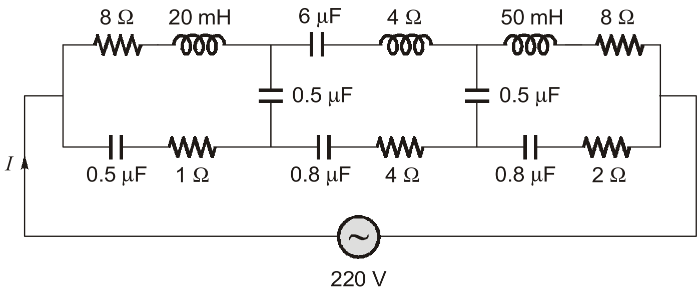
$$\frac{P+x}{P} = \frac{4 \times 3}{2} = 6$$

$$\Rightarrow \quad 1+\frac{x}{P} = 6$$

$$\frac{x}{P} = 5$$

$$x = 5P = 5 \times 4 = 20\ \Omega$$

Q.7 The effective current I in the given circuit at very high frequencies will be ________ A.

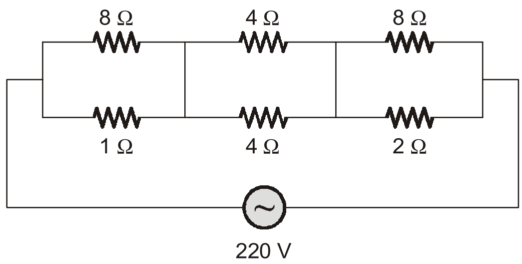


7. **(44)**

At very high frequencies,

$$X_c = \frac{1}{\omega C} \to 0$$

$$X_L = \omega L \to \infty$$

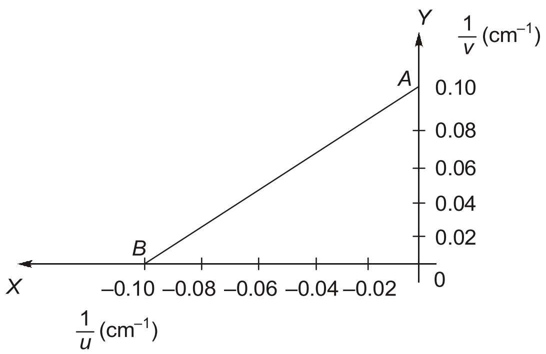


Thus equivalent circuit,

$$Z = 1 + 2 + 2 = 5\ \Omega$$

$$I = \frac{220}{5} = 44\ \text{A}$$

Q.8 The graph between $\frac{1}{u}$ and $\frac{1}{v}$ for a thin convex lens in order to determine its focal length is plotted as shown in the figure. The refractive index of lens is 1.5 and its both the surfaces have same radius of curvature R. The value of R will be ______ cm. (where u = object distance, v = image distance)



8. (10)

For point B, $\frac{1}{u} = -0.10\text{ cm}^{-1}$, $\frac{1}{v} = 0$

Thus, $u = -10$ cm, $v = \infty$

i.e., $f = 10$ cm

$$\frac{1}{10} = (1.5-1)\left(\frac{2}{R}\right) \Rightarrow \frac{1}{R} = \frac{1}{10}$$

$$R = 10\text{ cm}$$

Q.9 In the hydrogen spectrum, λ be the wavelength of first transition line of Lyman series. The wavelength difference will be "$a\lambda$" between the wavelength of 3rd transition line of Paschen series and that of 2nd transition line of Balmer series where a = ________.

9. (5)

For first line of Lyman, $$\frac{1}{\lambda} = R\left(1-\frac{1}{4}\right) = R\frac{3}{4}$$

$$\lambda = \frac{4}{3R}$$

3rd line (Paschen) $$\frac{1}{\lambda_3} = R\left(\frac{1}{3^2}-\frac{1}{6^2}\right) = \frac{R}{9}\times\frac{3}{4}$$

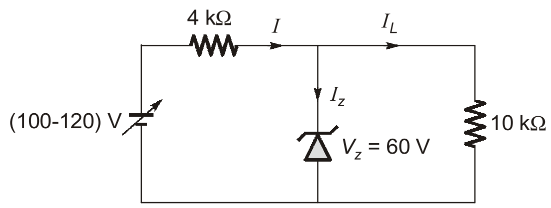
2nd Line (Balmer)

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = \frac{R}{4}\times\frac{3}{4}$$

Thus, $$a\lambda = \lambda_2 - \lambda_2 = \frac{12}{R} - \frac{16}{3R} = \frac{20}{3R}$$

$$a\left(\frac{4}{3R}\right) = \frac{20}{3R} \Rightarrow a = 5$$

Q.10 In the circuit shown below, maximum zener diode current will be _______ mA.



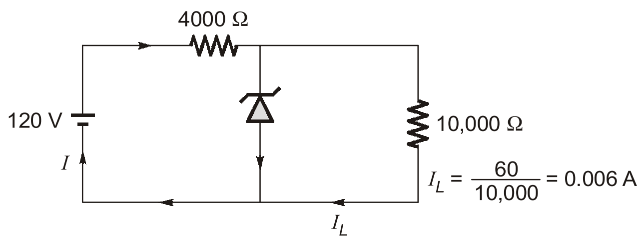
10. (9)

$$I = \frac{120-60}{4000} = 0.015\text{ A}$$

Thus, $$I_2 = I - I_L$$

$$= 0.015 - 0.006$$

$$= 0.009 = 9\text{ mA}$$



PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q.1 Match **List I** with **List II**:

List-I (Compound)	List-II (Shape)
A. BrF_5	(I) bent
B. $[CrF_6]^{3-}$	(II) Square pyramidal
C. O_3	(III) Trigonal bipyramidal
D. PCl_5	(IV) Octahedral

Choose the correct answer from the options given below:

(a) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(b) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
(c) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)
(d) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

1. (c)

BrF_5 – Square pyramid
$[CrF_6]^{3-}$ – Octahedral
O_3 – Bent
PCl_5 = Trigonal Bipyramid

Q.2 Match **List I** with **List II**:

List-I
(Processes/Reactions)

A. $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$
B. $4NH_3(g) + 5O_2(g) \rightarrow 4NO(g) + 6H_2O(g)$
C. $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$
D. Vegetable oil (l) + $H_2 \rightarrow$ Vegetable ghee(s)

List-II
(Catalyst)

(I) Fe(s)
(II) Pt(s) – Rh(s)
(III) V_2O_5
(IV) Ni(S)

Choose the correct answer from the options given below:

(a) (A)-(III), (B)-(I), (C)-(II),(D)-(IV)
(b) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)
(c) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(d) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

2. (b)

$2SO_2(g) + O_2(g) \xrightarrow{V_2O_5} 2SO_3(g)$ contact process

$4NH_3(g) + 5O_2(g) \xrightarrow{A(s)-Rh(s)} 4NO(g) + 6H_2O(g)$ ostwald's process

$N_2(g) + 3H_2(g) \xrightarrow{Fe(s)} 2NH_3(g)$; Haber's process

Vegetable oil (l) + $H_2 \xrightarrow{Ni(s)}$ Vegetable ghee; Hydrogenation

Q.3 Given two statements below:

Statement I : In Cl_2 molecules the covalent radius is double of the atomic radius of chlorine.

Statement II : Radius of anionic species is always greater than their parent atomic radius.

Choose the **most appropriate** answer from options given below:

(a) Both **Statement I** and **Statement II** are correct.

(b) Both **Statement I** and **Statement II** are incorrect.

(c) **Statement I** is correct but **Statement II** is incorrect.

(d) **Statement I** is incorrect but **Statement II** is correct.

3. (d)

Ionic radius of Cl^- = 167 pm

Covalne radius of Cl = 99 pm

Q.4 Refining using liquation method is the most suitable for metals with:

(a) Low melting point (b) High boiling point

(c) High electrical conductivity (d) Less tendency to be soluble in melts than impurities

4. (a)

Liquation process is used for purification of that metal, whose melting point is lesser then that of impurities.

Q.5 Which of the following can be used to prevent the decomposition of H_2O_2?

(a) Urea (b) Formaldehyde

(c) Formic acid (d) Ethanol

5. (a)

Urea acts as negative catalyst for decomposition of H_2O_2.

Q.6 Reaction of $BeCl_2$ with $LiAlH_4$ gives:

1. $AlCl_3$
2. BeH_2
3. LiH
4. LiCl
5. $BeAlH_4$

Choose the **correct** answer from options given below:

(a) 1, 4 and 5 (b) 1, 2 and 4

(c) 4 and 5 (d) 2, 3 and 4

6. (b)

$$2BeCl_2 + LiAlH_4 \longrightarrow 2BeH_2 + LiCl + AlCl_3$$

Q.7 Borazine, also known as inorganic benzene, can be prepared by the reaction of 3-equivalents of "*X*" with 6- equivalents of "*Y*". '*X*' and "*Y*", respectively are:

(a) $B(OH)_3$ and NH_3 (b) B_2H_6 and NH_3

(c) B_2H_6 and HN_3 (d) NH_3 and B_2O_3

7. (b)

$$B_2H_6 + NH_3 \xrightarrow{\text{High Temperature}} B_3N_3H_6 \text{(Borazine)}$$

Q.8 Which of the given reactions is not an example of disproportionation reaction?

(a) $2H_2O_2 \rightarrow 2H_2O + O_2$

(b) $2NO_2 + H_2O \rightarrow HNO_3 + HNO_2$

(c) $MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$

(d) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

8. **(c)**

It is reduction process, while rest are examples of disproportionation reaction.

Q.9 The dark purple colour $KMnO_4$ disappears in the titration with oxalic acid in acidic medium. The overall change in the oxidation number of manganese in the reaction is:

(a) 5 (b) 1

(c) 7 (d) 2

9. **(a)**

$\overset{+7}{Mn}O_4^- + 8H^+ + 5e \longrightarrow \overset{+2}{Mn^{2+}} + 4H_2O$ change in oxidation number of Mn = 7 – 2 = 5.

Q.10 $\dot{C}l + CH_4 \rightarrow A + B$

A and *B* in the above atmospheric reaction step are:

(a) C_2H_6 and Cl_2 (b) $\dot{C}HCl_2$ and H_2

(c) $\dot{C}H_3$ and HCl (d) C_2H_6 and HCl

10. **(c)**

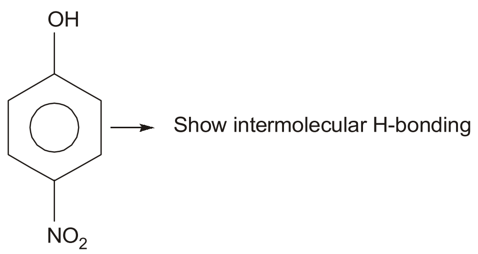
$\dot{C}l + CH_3 - H \longrightarrow \dot{C}H_3 + HCl$

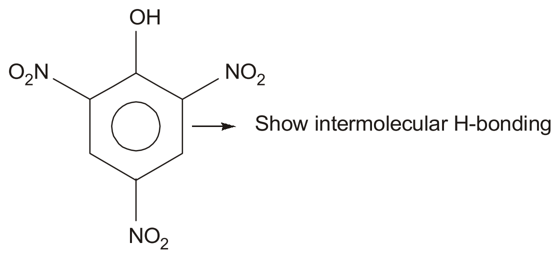
Q.11 Which technique among the following is most appropriate in separation of a mixture of 100 mg of p- nitrophenol and picric acid?

(a) Steam distillation (b) 2-5 ft long column of silica gel

(c) Sublimation (d) Preparative TLC (Thin layer Chromatography)

11. **(d)**





Solvent polarity has beam related to R_f value of nitro compounds 100 mg P-nitrophenol and picric acid have different R_f value on silica gel plate.

∴ Preparative TLC is best to separate 100 mg of P- nitro phenol and picric acid.

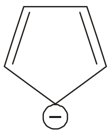
Q.12 The difference in the reaction of phenol with bromine in chloroform and bromine in water medium is due to:

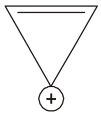
(a) Hyperconjugation in substrate
(b) Polarity of solvent
(c) Free radical formation
(d) Electromeric effect the substrate

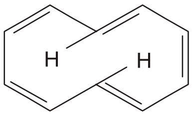
12. (b)

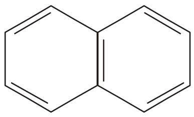
Water is more polar solvent than chloroform.

Q.13 Which of the following compounds in not aromatic?

(a) 

(b) 

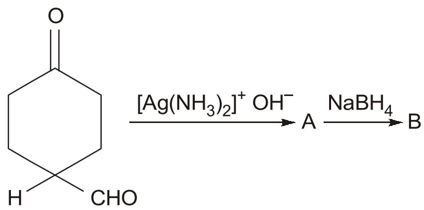
(c) 

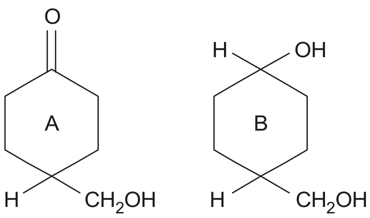
(d) 

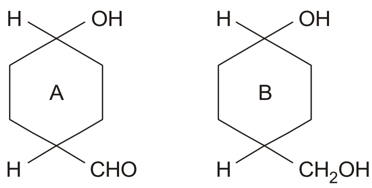
13. (c)

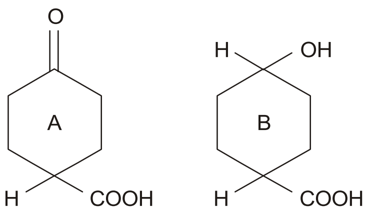
Both indicated hydrogen atoms repel each other so due steric hinderance the given compound becomes non planar. It is non-aromatic.

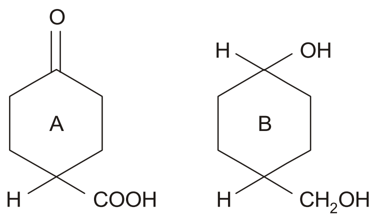
Q.14 The products formed in the following reaction, *A* and *B* are



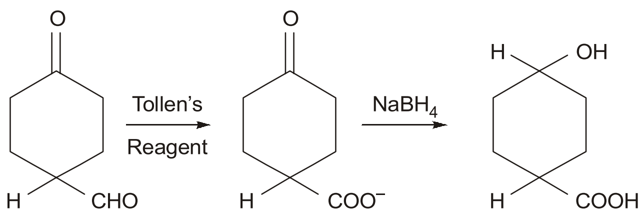
(a) 

(b) 

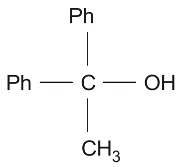
(c) 

(d) 

14. (c)



Q.15 Which reactant will give the following alcohol on reaction with one mole of phenyl magnesium bromide (PhMgBr) followed by acidic hydrolysis?



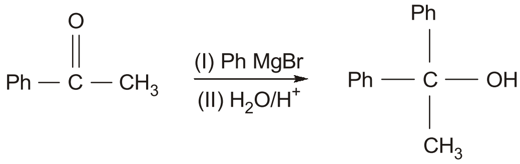
(a) $CH_3-C\equiv N$

(b) $Ph-C\equiv N$

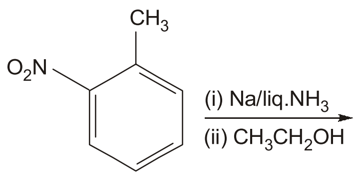
(c) $CH_3-\overset{O}{\overset{\|}{C}}-O-Ph$

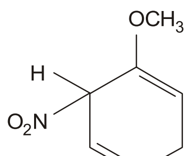
(d) $Ph-\underset{O}{\underset{\|}{C}}-CH_3$

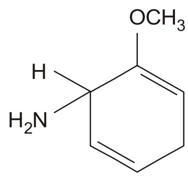
15. (d)

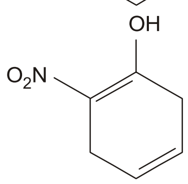


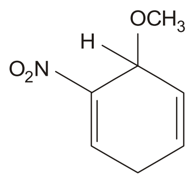
Q.16 The major product of the following reaction is



(a) 

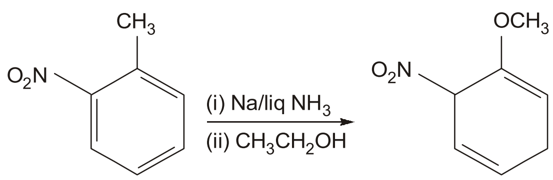
(b) 

(c) 

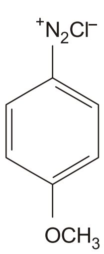
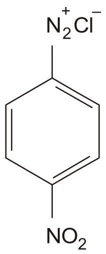
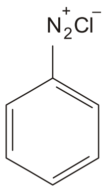
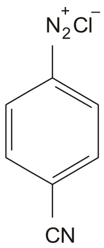
(d) 

16. (a)

Birch reduction takes place of carbon connected with electron withdrawing group ($-NO_2$)



Q.17 The correct stability order of the following diazonium salt is

1. 
2. 
3. 
4. 

(a) 1 > 2 > 3 > 4 (b) 1 > 3 > 4 > 2
(c) 3 > 1 > 4 > 2 (d) 3 > 4 > 2 > 1

17. (b)

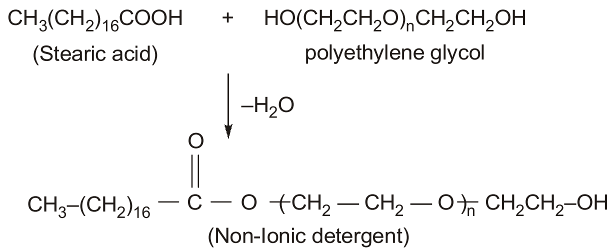
Electron donating group increases stability of diazonium salt. Whereas electron with drawing group decrease it.

$-OCH_3(+M)$, $-NO_2$ and $-CN(-M)$

Q.18 Stearic acid and polyethylene glycol react to form which one of the following soap/s detergents?

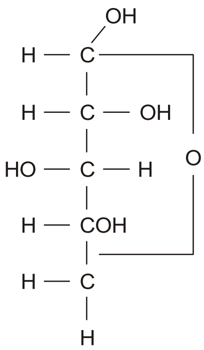
(a) Cationic detergent (b) Soap
(c) Anionic detergent (d) Non-ionic detergent

18. (d)

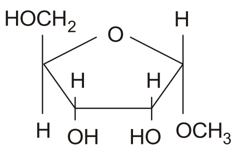


Q.19 Which one of the following is a reducing sugar?

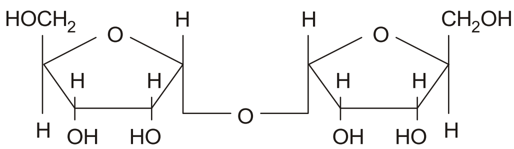
(a)



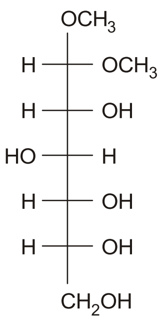
(b)



(c)



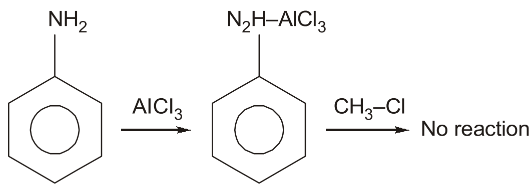
(d)



19. (a)
Reducing sugar must have hemi-acetal group and option (a) has that group.

Q.20 Given below are two statements : one is labeled as **Assertion (A)** and the other is labeled as **Reason (R)**.
Assertion (A) : Experimental reaction of CH_3Cl with aniline and anhydrous $AlCl_3$ does not give O and P-methylaniline.
Reason (R) : The $-NH_2$ groups of aniline becomes deactivating because of salt formation with anhydrous $AlCl_3$ and hence yields m-methyl aniline as the product.
In the light of the above statements, choose the **most appropriate** answer from the options given below:
(a) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**.
(b) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**.
(c) **(A)** is true, but **(R)** is false.
(d) **(A)** is false, but **(R)** is true.

20. (c)
Aniline makes adduct with $AlCl_3$ and thus gets deactivated for electrophilc aromatic substitution and do not react with CH_3-Cl.



(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q.1 Chlorophyll extracted from the crushed green leaves was dissolved in water to make 2 L solution of Mg of concentration 48 ppm. The number of atoms of Mg in this solution is $x \times 10^{20}$ atoms. The value of x is ___________. (Neatest Integer)
(Given : Atomic mass of Mg is 24 g mol^{-1}; $N_A = 6.02 \times 10^{23}$ mol^{-1})

1. **(24)**

$$\text{Mass of solute} = 48 \times 2 \times 10^{-3} \text{ gram}$$
$$\text{Atoms of Mg} = 24.08 \times 10^{20}$$

Q.2 A mixture of hydrogen and oxygen contains 40% hydrogen by mass when the pressure is 2.2 bar. The partial pressure of hydrogen is _________ bar. (Nearest Integer)

2. (2)

$$x_{H_2} = \frac{n_{H_2}}{n_{H_2} + n_{O_2}} = \frac{(40/2)}{(40/2)+(60/32)} = 0.914$$
$$P_{H_2} = x_{H_2} \times P_T = 0.914 \times 2.2 \sim 2 \text{ bar}$$

Q.3 The wavelength of an electron and a neutron will become equal when the velocity of the electron is x times the velocity of neutron. The value of x is _________. (Nearest Integer)
(Mass of electron is 9.1×10^{-31} kg and mass of neutron is 1.6×10^{-27} kg)

3. **(1758)**

$$Xe = Xn$$

$\Rightarrow$ $$MeVe = MnVn$$

$\Rightarrow$ $$\frac{Ve}{Vn} = \frac{Mn}{Me} \sim 1758.24$$

Q.4 2.4 g coal is burnt in a bomb calorimeter in excess of oxygen at 298 K and 1 atm pressure. The temperature of the calorimeter rises from 298 K to 300 K. The enthalpy change during the combustion of coal is $-x$ kJ mol^{-1}. The value of x is ____________. (Nearest Integer)
(Given : Heat capacity of bomb calorimeter 20.0 kJ K^{-1}. Assume coal to be pure carbon)

4. **(200)**

$C(s) + O_2(g) \longrightarrow CO_2(g)$ $\qquad \Delta ng = 0$

$$q_C = 20 \times 2 = 40 \text{ kJ}$$

$$\text{Number of moles of } C = \frac{2.4}{12} = 0.2 \text{ moles}$$

$$\Delta H_C = \frac{40 \text{ kJ}}{0.2 \text{ mole}} = 200 \text{ kJ/mole}$$

Q.5 When 800 mL of 0.5 M nitric acid is heated in a beaker, its volume is reduced to half and 11.5 g of nitric acid is evaporated. The molarity of the remaining nitric acid solution is $x \times 10^{-2}$ M. (Nearest Integer)
(Molar mass of nitric acid is 63 g mol^{-1})

5. **(54)**

Remaining volume of solution = 400 ml

$$\text{Mass of } HNO_3 = 25.2 - 11.5 = 13.7$$

$$\text{Molarity} = \frac{(13.7/63)}{(400/1000)} = 0.54 \text{ M} = 54 \times 10^{-2} \text{M}$$

Q.6 At 298 K, the equilibrium constant is 2×10^{15} for the reaction:

$$Cu(s) + 2Ag^+(aq) \rightleftharpoons Cu^{2+}(aq) + 2Ag(s)$$

The equilibrium constant for the reaction

$$\frac{1}{2}Cu^{2+}(aq) + Ag(s) \rightleftharpoons \frac{1}{2}Cu(s) + Ag^+(aq)$$

is $x \times 10^{-8}$. The value of x is ______________. (Nearest Integer)

6. **(2)**

$$K_{eq} = \left(\frac{1}{2 \times 10^{15}}\right)^{1/2} = 2.24 \times 10^{-8}$$

Q.7 The amount of charge in F (Faraday) required to obtain one mole of iron from Fe_3O_4 is________. (Nearest Integer)

7. **(3)**

$$Fe_3O_4 + 8H^4 + 8e \longrightarrow 3Fe + 4H_2O$$

Q.8 For a reaction $A \rightarrow 2B + C$ the half lives are 100 s and 50 s when the concentration of reactant A is 0.5 and 1.0 mol L^{-1} respectively. The order of the reaction is _____________. (Nearest Integer)

8. (2)

$t_{1/2} \alpha [\text{Reactant}]_{t=0}^{(1-n)}$, where n = order of reaction.

Q.9 The difference between spin only magnetic moment values of $[Co(H_2O)_6Cl_2$ and $Cr(H_2O)_6]Cl_3$ is __________.

9. (0)

$[Co(H_2O)_6]Cl_2 \Rightarrow Co^{2+} \Rightarrow 3d^7 \Rightarrow t_{2g}^5 eg^2$

Number of unpaired electrons = 3

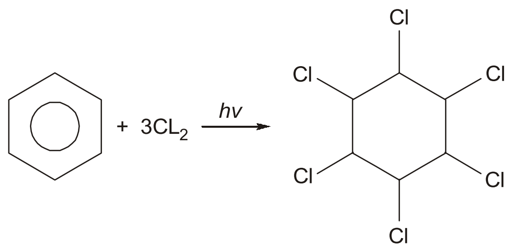
$[Cr(H_2O)_6]Cl_3 \Rightarrow Cr^{3+} \Rightarrow 3d^3 \Rightarrow t_{2g}^3 eg^0$

Number of unpaired electrons = 3

Due to same number of unpaired electrons, difference between spin only magnetic moment is zero.

Q.10 In the presence of sunlight, benzene reacts with Cl_2 to give product, *X*. The number of hydrogens in *X* is __________.

10. (6)



PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q.1 Let $f : R \to R$ be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$, then $f(14)$ is equal to :

(a) 4 (b) 10

(c) 11 (d) 16

1. **(b)**

$$f(3x) - f(x) = x$$

Replace $x \to \frac{x}{3} \Rightarrow f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3}$

Again replace $x \to \frac{x}{3} \Rightarrow f\left(\frac{x}{3}\right) - f\left(\frac{x}{3^2}\right) = \frac{x}{3^2}$

$$f\left(\frac{x}{3^{n-2}}\right) - f\left(\frac{x}{3^{n-1}}\right) = \frac{x}{3^{n-1}}$$

Adding all we get, $f(3x) - f\left(\frac{x}{3^{n-1}}\right) = x\left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\ldots \frac{1}{3^{n-1}}\right)$

$\therefore \lim_{n\to\infty}\left[f(3x) - f\left(\frac{x}{3^{n-1}}\right)\right] = \lim_{n\to\infty} x\left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots . \frac{1}{3^{n-1}}\right)$

$\Rightarrow f(3x) - f(0) = \frac{3x}{2}$

putting, $x = \frac{8}{3} \Rightarrow f(8) - f(0) = 4$

$\therefore f(0) = 3$

Also putting, $x = \frac{14}{3}$ in $f(3x) - 3 = \frac{3x}{2}$

$\Rightarrow f(14) - 3 = 7$

$\Rightarrow f(14) = 10$

Q.2 Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $\text{Re}(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true?

(a) $\arg z_2 = \pi - \tan^{-1} 3$ (b) $\arg(z_1 - 2z_2) = -\tan^{-1}\frac{4}{3}$

(c) $|z_2| = \sqrt{10}$ (d) $|2z_1 - z_2| = 5$

2. **(d)**

$$\frac{z_2 - 0}{(1+2i) - 0} = \left|\frac{OB}{OA}\right| e^{\frac{i\pi}{4}}$$

$\Rightarrow z_2 = (1 + 2i)(1 + i)$

$$= -1 + 3i$$

$\therefore$ $$\arg z_2 = \pi - \tan^{-1}3 \text{ and } |z_2| = \sqrt{10}$$

$$z_1 - 2z_2 = 3 - 4i$$

$\therefore$ $$\arg(z_1 - 2z_2) = -\tan^{-1}\frac{4}{3}$$

$\Rightarrow$ $$|z_1 - 2z_2| = |2 + 4i + 1 - 3i| = \sqrt{10}$$

Q.3 If the system of linear equations.

$8x + y + 4z = -2$

$x + y + z = 0$

$\lambda x - 3y = \mu$

has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane $8x + y + 4z + 2 = 0$ is:

(a) $3\sqrt{5}$ (b) 4

(c) $\frac{26}{9}$ (d) $\frac{10}{3}$

3. (d)

$$\Delta = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix} = 12 - 3\lambda$$

So for $\lambda = 4$, it is having infinitely many solutions.

$$\Delta x = \begin{vmatrix} -2 & 1 & 4 \\ 0 & 1 & 1 \\ \mu & -3 & 0 \end{vmatrix}$$

$$= -6 - 3\mu = 0$$

$\Rightarrow$ $$-6 - 3\mu = 0$$

For $\mu = -2$ Distance of $\left(4, -2, -\frac{1}{2}\right)$ from $8x + y + 4z + 2 = 0 \left|\frac{32 - 2 - 2 + 2}{\sqrt{64 + 1 + 16}}\right| = \frac{10}{3}$units

Q.4 Let A be a 2×2 matrix with det $(A) - 1$ and $\det((A + I)(\text{Adj}(A) + I)) = 4$. Then the sum of the diagonal elements of A can be :

(a) -1 (b) 2

(c) 1 (d) $-\sqrt{2}$

4. (b)

$$|(A + I)(\text{adj } A + I)| = 4$$

$\Rightarrow$ $$|A \text{ adj } A + A + \text{Adj } A + I| = 4$$

$\Rightarrow$ $$|(A)I + A + \text{adj } A + I| = 4|A| = -1 \Rightarrow |A + \text{adj } A| = 4$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \quad \text{adj } A = \begin{bmatrix} a & -b \\ -c & d \end{bmatrix} \Rightarrow \begin{vmatrix} (a+d) & 0 \\ 0 & (a+d) \end{vmatrix} = 4$$

$$\Rightarrow \quad a + d = \pm 2$$

Q.5 The odd natural number a, such that the area of the region bounded by $y = 1$, $y = 3$, $x = 0$, $x = y^a$ is $\frac{364}{3}$, is equal to:

(a) 3 (b) 5

(c) 7 (d) 9

5. **(b)**

Since a is a odd natural number than

$$\left|\int_1^3 y^a dy\right| = \frac{364}{3}$$

$$\Rightarrow \quad \left|\left(\frac{y^{a+1}}{a+1}\right)_1^3\right| = \frac{364}{3} \Rightarrow \frac{3^{a+1}-1}{a+1} = \frac{364}{3} \quad \Rightarrow \quad a = 5$$

Q.6 Consider two G.P.'s 2, 2^2, 2^3, ... and 4, 4^2, 4^3, ... of 60 and n terms respectively. If the geometric mean of all the 60 + n terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to:

(a) 560 (b) 1540

(c) 1330 (d) 2600

6. **(c)**

Given G.P's 2, 2^2, 2^3, ...60 term and 4, 4^2, 4^3, ... of 60.

Now $\quad$ G.M. $= (2)^{\frac{225}{8}}$

$$\Rightarrow \quad (2, 2^2, 2^3, \ldots)^{\frac{1}{60+n}} = (2)^{\frac{225}{8}} \quad \Rightarrow \quad n = \frac{57}{8}, 29 \text{ so } n = 20$$

$$\therefore \sum_{k=1}^{\infty} k(n-k) 20 \times \frac{20 \times 21}{2} - \frac{20 \times 21 \times 41}{6} = 1330$$

Q.7 If the function $f(x) = \begin{cases} \frac{\log_e(1 - x + x^2) + \log_e(1 + x + x^2)}{\sec x - \cos x}, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) - \{0\} \end{cases}$ is continuous at $x = 0$, then k is equal to

(a) 1 (b) –1

(c) e (d) 0

7. **(a)**

$$f(x) = \begin{cases} \frac{\log_e(1 + x + x^2) + \log_e(1 + x + x^2)}{\sec x - \cos x}, & x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) - \{0\} \\ k \quad , & x = 0 \end{cases} \text{ for continuity at } x = 0$$

$$\lim_{x \to 0} f(x) = k$$

$\therefore$ $$k = \lim_{x\to 0}\frac{\log_e(1+x^2+x^4)}{\sec x - \cos x}\left(\frac{0}{0}\text{form}\right)$$

$$= \lim_{x\to 0}\frac{\cos x \log_e(1+x^2+x^4)}{\sin^2 x} = 1$$

Q.8 If $f(x) = \begin{cases} x+a, & x \le 0 \\ |x-4|, & x > 0 \end{cases}$ and $g(x) = \begin{cases} x+1, & x < 0 \\ (x-4)^2 + b, & x \ge 0 \end{cases}$ are continuous on R, then (gof)(2) + (fog)(–2) is equal to:

(a) –10 (b) 10

(c) 8 (d) –8

8. (d)

$f(x) = \begin{cases} x+a, & x \le 0 \\ |x-4|, & x > 0 \end{cases}$ and $g(x) = \begin{cases} x+1, & x < 0 \\ (x-4)^2 + b, & x \ge 0 \end{cases}$

$\because$ $f(x)$ and $g(x)$ are continuous on R

$\therefore$ $a = 4$ and $b = 1 - 16 = -15$

then $(gof)(2) + (fog)(-2) = g(2) + f(-1)$

$= -11 + 3 = -8$

Q.9 If $f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \le 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases}$. Then the set of all values of b, for which $f(x)$ has maximum value at $x = 1$, is :

(a) $(-6, -2)$ (b) $(2, 6)$

(c) $[-6-2) \cup (2, 6]$ (d) $[-\sqrt{6}-2) \cup (2, \sqrt{6}]$

9. (c)

$$f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \le 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases}$$

If $f(x)$ has maximum value at $x = 1$ then

$f(1^+) \le f(1)$

$\Rightarrow$ $-2 + \log_2(b^2 - 4) \le 1 - 1 + 10 - 7$

$\Rightarrow$ $\log_2(b^2 - 4) \le 5$

$\Rightarrow$ $0 < b^2 - 4 \le 32$

$\Rightarrow$ $b^2 - 4 > 0 \Rightarrow b \in (-\infty, -2) \cup (2, \infty)$ ...(i)

And $b^2 - 4 \le 32 \Rightarrow b \in [-6, 6]$ ...(ii)

From (i) and (ii) we get $b \in [-6, -2) \cup (2, 6]$

Q.10 If $a = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{2n}{n^2+k^2}$ and $f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}}$, $x \in (0, 1)$, then:

(a) $2\sqrt{2}f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$ (b) $f\left(\frac{a}{2}\right)f'\left(\frac{a}{2}\right) = \sqrt{2}$

(c) $\sqrt{2}f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$ (d) $f\left(\frac{a}{2}\right) = \sqrt{2}f'\left(\frac{a}{2}\right)$

10. (c)

$$a = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{2n}{n^2+k^2} = \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{2}{1+\left(\frac{k}{n}\right)^2}$$

$$\therefore \quad a = \int_0^1 \frac{2}{1+x^2}dx = 2\tan^{-1}x\Big]_0^1 = \frac{\pi}{2}$$

$$f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}},\ x \in (0, 1)$$

$$\Rightarrow \quad f(x) = \frac{1-\cos x}{\sin x} = \operatorname{cosec} x - \cot x$$

$$\Rightarrow \quad f'(x) = \operatorname{cosec}^2 x - \operatorname{cosec} x \cot x \Rightarrow \left\{\begin{array}{l} f\left(\frac{a}{2}\right) = f\left(\frac{\pi}{4}\right) = \sqrt{2}-1 \\ f'\left(\frac{a}{2}\right) = f'\left(\frac{\pi}{4}\right) = 2-\sqrt{2} \end{array}\right.$$

$$\Rightarrow \quad f'\left(\frac{a}{2}\right) = \sqrt{2}f\left(\frac{a}{2}\right)$$

Q.11 If $\frac{dy}{dx} + 2y\tan x = \sin x,\ 0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of $y(x)$ is:

(a) $\frac{1}{8}$ (b) $\frac{3}{4}$

(c) $\frac{1}{4}$ (d) $\frac{3}{8}$

11. (a)

$$\frac{dy}{dx} + 2y\tan x = \sin x,$$

$$\therefore \quad \text{I.F. } e^{\int 2\tan x\, dx} = \sec^2 x$$

$$\Rightarrow \quad y\sec^2 x = \sec x + c$$

$$\Rightarrow \quad y\left(\frac{\pi}{3}\right) = 0$$

$$\Rightarrow \quad c = -2$$

$$\Rightarrow \quad y\sec^2 x = \sec x - 2$$

$$\Rightarrow \quad y = \frac{\sec x - 2}{\sec^2 x}$$

$$\Rightarrow \quad \cos x - 2\cos^2 x = -2\left(\cos x - \frac{1}{4}\right)^2 + \frac{1}{8}$$

$$\therefore \quad y_{\max} = \frac{1}{8}$$

Q.12 A point P moves so that the sum of squares of its distances from the points (1, 2) and (–2, 1) is 14. Let $f(x, y) = 0$ be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the points C, D. Then the area of the quadrilateral $ACBD$ is equal to :

(a) $\frac{9}{2}$ (b) $\frac{3\sqrt{17}}{2}$

(c) $\frac{3\sqrt{17}}{4}$ (d) 9

12. (b)

Let point $P : (h, k)$

Therefore according to question,

$(h-1)^2 + (k-2)^2 + (h+2)^2 + (k-1)^2 = 14$

$\therefore$ locus of $P(h, k)$ is $x^2 + y^2 + x - 3y - 2 = 0$

Now intersection with x-axis are $x^2 + x - 2 = 0 \Rightarrow x = -2, 1$

Now intersection with y-axis are $y^2 - 3y - 2 = 0 \Rightarrow y = \frac{3 \pm \sqrt{17}}{2}$

Therefore area of the quadrilateral $ABCD$ is $= \frac{1}{2}(|x_1| + |x_2|)(|y_1| + |y_2|)$

$$= \frac{1}{2} \times 3 \times \sqrt{17} = \frac{3\sqrt{17}}{2}$$

Q.13 Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α, β) is perpendicular to the line $2x + 2y = 5$. Then the normal to the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at the point $(\alpha + 4, \beta + 4)$ does NOT pass through the point:

(a) (25, 10)
(b) (20, 12)
(c) (30, 8)
(d) (15, 13)

13. (d)

Any tangent to $y^2 = 24x$ at (α, β) is $\beta y = 12(x + \alpha)$

therefore, slope $= \frac{12}{\beta}$

and perpendicular to $2x + 2y = 5$

$\Rightarrow$ $12 = \beta$ and $\alpha = 6$.

Hence hyperbola is $\frac{x^2}{6^2} - \frac{y^2}{12^2} = 1$ and normal is drawn at (10, 16)

therefore equation of normal

$$\frac{36x}{10} + \frac{144y}{16} = 36 + 144$$

$$\Rightarrow \quad \frac{x}{50} + \frac{y}{20} = 1$$

This does not pass though (15, 13) out of given option.

Q.14 The length of the perpendicular from the point (1, –2, 5) on the line passing through (1, 2, 4) and parallel to the line $x + y - z = 0 = x - 2y + 3z - 5$ is:

(a) $\sqrt{\frac{21}{2}}$
(b) $\sqrt{\frac{9}{2}}$
(c) $\sqrt{\frac{73}{2}}$
(d) 1

14. (a)

The line $x + y - z = 0 = x - 2y + 3z - 5$ is parallel to the vector

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = (1, 4, -3)$$

Equation of line through P(1, 2, 4) and parallel to

$$\vec{b}\frac{x-1}{1} = \frac{y-2}{-4} = \frac{z-4}{-3}$$

Let $N \equiv (\lambda + 1, -4\lambda + 2, -3\lambda + 4)$

$$\overline{QN} = (\lambda, -4\lambda + 4, -3\lambda - 1)$$

$\overline{QN}$ is perpendicular to $\vec{b}$ $\Rightarrow$ $(\lambda, -4\lambda + 4, -3\lambda - 1) \times (1, 4, -3) = 0$ $\Rightarrow$ $\lambda = \frac{1}{2}$

Hence, $\overline{QN}\left(\frac{1}{2}, 2, \frac{-5}{2}\right)$ and $|\overline{QN}| = \sqrt{\frac{21}{2}}$

Q.15 Let $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}, \alpha > 0.$ If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30, then α is equal to:

(a) $\frac{15}{2}$ (b) 8

(c) $\frac{13}{2}$ (d) 7

15. (d)

Given: $\vec{a} = (\alpha, 1, -1)$ and $\vec{b} = (2, 1, -\alpha)\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & -1 \\ 2 & 1 & -\alpha \end{vmatrix}$

$$= (-\alpha + 1)\hat{i} + (\alpha^2 - 2)\hat{j} + (\alpha - 2)\hat{k}$$

Projection of $\vec{c}$ on $\vec{d}$ $= -\hat{i} + +2\hat{j} - 2\hat{k}$

$$= \left|\vec{c} \cdot \frac{\vec{d}}{|\vec{d}|}\right| = 30 \text{ \{Given\}}$$

$$\Rightarrow \left|\frac{\alpha - 1 - 4 + 2\alpha^2 - 2\alpha + 4}{\sqrt{1 + 4 + 4}}\right| = 30$$

On solving $\Rightarrow$ $\alpha = \frac{-13}{2}$ (Rejected as $\alpha > 0$) and $\alpha = 7$

Q.16 The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X = 1) = \frac{4}{243}$, then $P(X = 4 \text{ or } 5)$ is equal to:

(a) $\frac{5}{9}$ (b) $\frac{64}{81}$

(c) $\frac{16}{27}$ (d) $\frac{145}{243}$

16. (c)

Given, mean $= np = \alpha$, and variance $= npq = \frac{\alpha}{3}$ $\Rightarrow$ $q = \frac{1}{3}$ and $p = \frac{2}{3}$

$$P(X = 1) = np^1q^{n-1} = \frac{4}{243}$$

$$\Rightarrow \quad n\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^{n-1} = \frac{4}{243}$$

$$\Rightarrow \quad n = 6$$

$$\Rightarrow \quad P(X = 4 \text{ or } 5) = {}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1 = \frac{16}{27}$$

Q.17 Let E_1, E_2, E_3 be three mutually exclusive such that $P(E_1) = \frac{2+3p}{6}$, $P(E_2) = \frac{2-p}{8}$ and $P(E_3) = \frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(p_1 + p_2)$ is equal to:

(a) $\frac{2}{3}$ (b) $\frac{5}{3}$

(c) $\frac{5}{4}$ (d) 1

17. (b)

$$0 \le \frac{2+3p}{6} \le 1$$

$$\Rightarrow \quad p \in \left[-\frac{2}{3}, \frac{4}{3}\right],$$

$$0 \le \frac{2-p}{8} \le 1$$

$$\Rightarrow \quad p \in [-6, 2]$$

and
$$0 \le \frac{1-p}{2} \le 1$$

$$\Rightarrow \quad p \in [-1, 1]$$

$$0 < P(E_1) + P(E_2) + P(E_3) \le 10 \le \frac{13}{12} - \frac{p}{8} \le 1$$

$$\Rightarrow \quad p \in \left[\frac{2}{3}, \frac{26}{3}\right]$$

Taking intersection to all
$$p \in \left[\frac{2}{3}, 1\right]$$

$$\therefore \quad p_1 + p_2 = \frac{5}{3}$$

Q.18 Let $S = \left\{\theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\right\}$. Then $n(s) + \sum_{\theta \in S}\left(\sec\left(\frac{\pi}{4} + 2\theta\right)\text{cosec}\left(\frac{\pi}{4} + 2\theta\right)\right)$ is equal to :

(a) 0 (b) –2

(c) –4 (d) 12

18. (c)

$$S = \left\{\theta \in [0, 2\pi] : 8^{2\sin^2 x} + 8^{2\cos^2 x} = 16\right\}$$

Now apply AM ≥ GM for $\frac{8^{2\sin^2 x} + 8^{2\cos^2 x}}{2} \ge (8^{2\sin^2 x + 2\cos^2 x})^{\frac{1}{2}}$

$$\Rightarrow \quad 8^{2\sin^2 x} = 8^{2\cos^2 x}$$

$$\Rightarrow \quad \sin^2\theta = \cos^2\theta$$

$\therefore \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$$n(S) + \sum_{\theta \in S}\left(\sec\left(\frac{\pi}{4} + 2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right)\right) = 4 + \sum_{\theta \in S}\left\{\frac{2}{2\sin\left(\frac{\pi}{4} + 2\theta\right)\cos\left(\frac{\pi}{4} + 20\right)}\right\}$$

$$= 4 + \sum_{\theta \in S}\left(\frac{2}{\sin\left(\frac{\pi}{2} + 4\theta\right)}\right) = 4 + 2\sum_{\theta \in S}\left(\operatorname{cosec}\left(\frac{\pi}{2} + 4\theta\right)\right)$$

$$= 4 + \left[\operatorname{cosec}\left(\frac{\pi}{2} + \pi\right) + \operatorname{cosec}\left(\frac{\pi}{2} + 3\pi\right) + \operatorname{cosec}\left(\frac{\pi}{2} + 5\pi\right) + \operatorname{cosec}\left(\frac{\pi}{2} + 7\pi\right)\right]$$

$$= 4 - 2(4) = -4$$

Q.19 $\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$ is equal to :

(a) 1 (b) 2

(c) $\frac{1}{4}$ (d) $\frac{5}{4}$

19. (b)

$$\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)\tan\left(2\tan^{-1}\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\times\frac{1}{8}} + \sec^{-1}\frac{\sqrt{5}}{2}\right)$$

$$= \tan\left(2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}\right) = \tan\left(\tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}} + \tan^{-1}\frac{1}{2}\right)$$

$$= \tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{2}\right) = \tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{8}}\right)$$

$$= \tan\left(\tan^{-1}\frac{\frac{5}{4}}{\frac{5}{8}}\right) = 2$$

Q.20 The statement $(-(p \Leftrightarrow \sim q)) \wedge q$ is:

(a) a tautology (b) a contradiction

(c) equivalent to $(p \Rightarrow q) \wedge q$ (d) equivalent to $(p \Rightarrow q) \wedge p$

20. (d)

$\sim(p \Leftrightarrow \sim q) \wedge q = (p \Leftrightarrow \sim q) \wedge q$ is:

p	q	$p \leftrightarrow q$	$(p \leftrightarrow q) \wedge q$	$(p \to q)$	$(p \to q) \wedge q$	$(p \to q) \wedge p$
T	T	T	T	T	T	T
T	F	F	F	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	F	F

$\therefore$ $(\sim (p \Leftrightarrow \sim q)) \wedge q$ is equivalent to $(p \Rightarrow q) \wedge p$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q.1 If for some $p, q, r \in R$, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2 + r^2}{p^2}$ is equal to ______.

1. (272)

Let α and β are the roots of

$$(p^2 + q^2)x^2 - 2q(p + r) + q^2 + r^2 = 0$$

$\therefore \quad \alpha + \beta > 0$ and $\alpha\beta > 0$.

Also, it has a common root with

$$x^2 + 2x - 8 = 0$$

$\therefore$ The common root between above two equations is 4.

$\Rightarrow \quad 16(p^2 + q^2) - 8q(p + r) + q^2 + r^2 = 0 \quad \Rightarrow \quad (16p^2 - 8pq + q^2) + (16q^2 - 8qr + r^2) = 0$

$\Rightarrow \quad (4p - q)^2 + (4q - r)^2 = 0$

$\Rightarrow \quad q = 4p$

and $\quad r = 16p$

$$\therefore \quad \frac{q^2 + r^2}{p^2} = \frac{16p^2 + 256p^2}{p^2} = 272$$

Q.2 The number of 5-digit natural numbers, such that the product of their digits is 36, is ________.

2. (180)

Factors of $36 = 2^2 \cdot 3^2 \cdot 1$

Five-digit combinations can be

(1, 2, 2, 3, 3) ,(1, 4, 3, 3, 1), (1, 9, 2, 2, 1),(1, 4, 9, 11) (1, 2, 3, 6, 1), (1, 6, 6, 1, 1)

i.e. total numbers

$$\frac{5!\,5!}{2!\,2!} + \frac{5!}{2!\,2!} + \frac{5!}{2!\,2!} + \frac{5!}{3!} + \frac{5!}{2!} + \frac{5!}{3!\,2!} = (30 \times 3) + 20 + 60 + 10 = 180$$

Q.3 The series of positive multiples of 3 is divided into sets : {3}, {6, 9 ,12}, {15, 18, 21, 24, 27}, ... Then the sum of the elements in the 11^{th} set is equal to ________.

3. (6993)

Given series $\{3 \times 1\}, \{3 \times 2, 3 \times 3, 3 \times 4\}, \{3 \times 5, 3 \times 6, 3 \times 7, 3 \times 8, 3 \times 9\}$......

$\therefore \quad 11^{th}$ set will have $1 + (10)2 = 21$ terms

Also up to 10th set total $3 \times k$ type terms will be $1 + 3 + 5 + ... + 19 = 100$ terms

$\therefore \quad$ Set 11 $= \{3 \times 101, 3 \times 102, ...3 \times 121\}$

$\therefore \quad$ Sum of elements $= 3 \times (101 + 102 + ... + 121)$

$$= \frac{3 \times 222 \times 21}{2} = 6993$$

Q.4 The number of distinct real roots of the equation

$x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is ......

4. (3)

$x^8 - x^7 + x^6 + x^5 + 3x^4 - 4x^3 - 2x^2 + 4x - 1 = 0$

$\Rightarrow$ $x^7(x-1) - x^5(x-1) + 3x^3(x-1) - x(x^2-1) + 2x(1-x) + (x-1) = 0$

$\Rightarrow$ $(x-1)(x^7 - x^5 + 3x^3 - x(x+1) - 2x + 1) = 0$

$\Rightarrow$ $(x-1)(x^7 - x^5 + 3x^3 - x^2 - 3x + 1) = 0$

$\Rightarrow$ $(x-1)(x^5(x^2-1) + 3x(x^2-1) - 1(x^2-1)) = 0$

$\Rightarrow$ $(x-1)(x^2-1)(x^5 + 3x - 1) = 0$

$\therefore$ $x = \pm 1$ are roots of above equation and $x^5 + 3x - 1$ is a monotonic term hence vanishes at exactly one value of x other then 1 or –1.

$\therefore$ 3 real roots.

Q.5 If the coefficients of x and x^2 in the expansion of $(1+x)^p(1-x)^q$, $p, q \le 15$, are –3 and –5 respectively, then the coefficient of x^3 is equal to ________.

5. (23)

Coefficient of x in $(1+x)^p(1-x)^q = -{}^pC_0\,{}^qC_1 + {}^pC_1\,{}^qC_0$

$= -3$

$\Rightarrow$ $p - q = 3$

Coefficient of x^2 in $(1+x)^p(1-x)^q = -{}^pC_0\,{}^qC_2 - {}^pC_1\,{}^qC_1 - {}^pC_2\,{}^qC_0$

$= -5$

$$\Rightarrow \quad \frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} = -5$$

$$\Rightarrow \quad \frac{q(q-1)}{2} - (q-3)q + \frac{(q-3)(q-4)}{2} = -5$$

$\Rightarrow$ $q = 11,\ p = 8$

Coefficient of x^3 in $(1+x)^8(1-x)^{11} = -{}^{11}C_3 + {}^8C_1\,{}^{11}C_2 - {}^8C_2\,{}^{11}C_1 + {}^8C_3$

$= 23$

Q.6 If $n(2n+1)\int_0^1 (1-x^n)^{2n}dx = 1177\int_0^1 (1-x^n)^{2n+1}dx,$ then $n \in N$ is equal to _______.

6. (24)

$\int_0^1 1\cdot(1-x^n)^{2n-1}dx$ using by parts we get

$$(2n^2 + n + 1)\int_0^1 (1-x^n)^{2n+1}dx = 1177\int_0^1 (1-x^n)^{2n+1}dx$$

$\Rightarrow$ $2n^2 + n + 1 = 1177$

$\Rightarrow$ $n = 24$ or $-\frac{49}{2}$

$\therefore$ $n = 24$

Q.7 Let a curve $y = y(x)$ pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and $x (> 3)$ be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $\left(\alpha, 6\sqrt{10}\right)$ in the first quadrant, then α is equal to ________

7. (6)

$$\int_3^x f(x)dx = \left(\frac{f(x)}{x}\right)^3$$

$$\Rightarrow \quad x^3\int_3^x f(x)fx = f^3(x), \text{ differentiating w.r. to } x$$

$$x^2 f(x) + 3x^2\frac{f^3(x)}{x^3} = 3f^2(x)f'(x)$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} = x^3y + \frac{3y^3}{x}$$

$$\Rightarrow \quad 3xy\frac{dy}{dx} = x^4 + 3y^2$$

After solving we get $y^2 = \frac{x^4}{3} + cx^2$ also curve passes through (3, 3) $\Rightarrow c = -2$

$$\therefore \quad y^2 = \frac{x^4}{3} - 2x^2 \text{ which passes through } (\alpha, 6\sqrt{10})$$

$$\therefore \quad \frac{\alpha^4 - 6\alpha^2}{3} = 360 \quad \Rightarrow \quad \alpha = 6$$

Q.8 The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 15a$ and $x - y = 3$ respectively. If its orthocenter is $(2, a), -\frac{1}{2} < a < 2,$ then p is equal to ________

8. (3)

$$\text{Slope of } AH = \frac{a+2}{1}$$

$$\text{Slope of } BC = -\frac{1}{p}$$

$$\therefore \quad p = a + 2$$

$$\therefore \quad C\left(\frac{18p-30}{p+1}, \frac{15p-33}{p+1}\right)$$

$$\text{Slope of } HC = \frac{16p - p^2 - 31}{16p - 32}$$

Slope of $BC \times$ slope of $HC = -1$

$$\Rightarrow \quad p = 3 \text{ or } 5$$

Hence, $p = 3$ is only possible value.

Q.9 Let the function $f(x) = 2x^2 - \log_e x$, $x > 0$, be decreasing in $(0, a)$ and increasing in $(a, 4)$. A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point $(8a, 8a - 1)$ but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then $\alpha + \beta$ is equal to _______.

9. (45)

$$f'(x) = \frac{4x^2 - 1}{x}$$

so $f(x)$ is decreasing in $\left(0, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \infty\right)$ $\Rightarrow$ $a = \frac{1}{2}$

Tangent at $y^2 = 2x$ is $y = mx + \frac{1}{2m}$ it is passing through (4, 3) therefore we get $m = \frac{1}{2}$ or $\frac{1}{4}$

So tangent may be $y = \frac{1}{2}$ or $\frac{1}{4}$

or $y = \frac{1}{4}x + 2$

but $y = \frac{1}{2}x + 1$ passes through (–2, 0) so rejected.

Equation of normal $\frac{x}{9} + \frac{y}{36} = 1$

Q.10 Let Q and R be two points on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point $P(4, 2, 7)$. Then the square of the area of the triangle PQR is _________.

10. **(153)**

Let PT perpendicular to QR

$$\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2} = \lambda$$

$\Rightarrow$ $T(2\lambda - 1, 3\lambda - 2, 2\lambda + 1)$ therefore

$2(2\lambda - 5) + 3(3\lambda - 4) + 2(2\lambda - 6) = 0 \quad \Rightarrow \quad \lambda = 2$

$T(3,4, 5)$

$\therefore \quad PT = \sqrt{1+4+4} = 3$

$\therefore \quad QT = \sqrt{26-9} = \sqrt{17}$

$\therefore \quad \Delta PQR = \frac{1}{2} \times 2\sqrt{17} \times 3 = 3\sqrt{17}$

Therefore square of ar(ΔPQR) = 153

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