MADE EASY & NEXT IAS GROUP

PRESENT

MENNIT

NEET | IIT-JEE | FOUNDATION

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JEE (MAIN) 2022

Test Date: 25th July 2022 (First Shift)

PAPER-1

Time Allotted: 3 Hours **Maximum Marks: 300**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

IMPORTANT INSTRUCTIONS:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts**. **Part-A** is *Physics*, **Part-B** is *Chemistry* and **Part-C** is *Mathematics*. Each part has only two sections: **Section-A** and **Section-B**.
4. **Section – A :** Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** contains 20 multiple choice questions which have only one correct answer. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (1 – 10)** contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q.1 If momentum [*P*], area [*A*] and time [*T*] are taken as fundamental quantities, then the dimensional formula for coefficient of viscosity is :

(a) $[PA^{-1}T^0]$ (b) $[PAT^{-1}]$

(c) $[PA^{-1}T]$ (d) $[PA^{-1}T^{-1}]$

1. (a)

$$\text{Viscosity} = \text{Pascal . Second}$$
$$P^X A^Y T^Z = [M^1 L^{-1} T^{-1}]$$
$$= [M^1 L^{+1} T^{-1}]^x [L^2]^y [T^1]^z = M^{-1} L^{-1} T^{-1}$$
$$= M^x L^{x+2y} T^{-x+z} = M^1 L^{-1} T^{-1}$$
$$= x = 1, \quad x + 2y = -1, \quad -x + z = 1$$
$$y = -1, \quad z = 0$$
$$\text{Viscosity} = [P^1 A^{-1} T^0]$$

Q.2 Which of the following physical quantities have the same dimensions?

(a) Electric displacement $(\vec{D})$ and surface charge density

(b) Displacement current and electric filed

(c) Current density and surface charge density

(d) Electric potential and energy

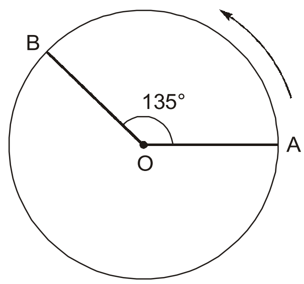
2. (a)

Electric Displacement

$$\vec{D} = \epsilon_0 \vec{E}$$
$$[D] = \left[\epsilon_0 \vec{E}\right] = \left[\epsilon_0 \frac{\sigma}{\epsilon_0}\right]$$
$$[D] = [\sigma]$$

Electric displacement ($\vec{D}$) and surface change density

Q.3 A person moved from *A* to *B* on a circular path as shown in figure. If the distance travelled by him is 60 m, then the magnitude of displacement would be:



(Given cos135° = – 0.7)

(a) 42 m (b) 47 m

(c) 19 m (d) 40 m

3. (b)

$$d = R\theta$$
$$60 = R\left[\frac{3\pi}{4}\right]$$

$$R = \frac{60 \times 4}{3\pi} = \frac{80}{\pi} m$$

$$\text{Displacement} = \sqrt{R^2 + R^2 - 2R^2 \cos 135}$$

$$= \sqrt{2R^2 - 2R^2(-0.7)}$$

$$= \sqrt{3.4R^2}$$

$$= \sqrt{3.4\left(\frac{80}{4}\right)^2}$$

$$\approx 47 \text{ m}$$

Q.4 A body of mass 0.5 kg travels on straight line path velocity $u = (3x^3 + 4)$ m/s. The net work done by the force during its displacement from $x = 0$ to $x = 2$ m is :

(a) 64 J (b) 60 J
(c) 120 J (d) 128 J

4. **(b)**

$$v_i = 3 \times (0)^2 + 4 = 4 \text{ m/s}$$

$$v_F = 3 \times 2^2 + 4 = 16 \text{ m/s}$$

$$\omega = \Delta k = \frac{1}{2} m \left[v_F^2 - v_i^2\right]$$

$$= \frac{1}{2} \times 0.5 \left[16^2 - 4^2\right] = \frac{240}{4} = 60 \text{ J}$$

Q.5 A solid cylinder and a solid sphere, having same mass M and radius R, roll down the same inclined plane from top without slipping. They start from rest. The ratio of velocity of the solid cylinder to that of the solid sphere, with which they the ground, will be:

(a) $\frac{\sqrt{5}}{3}$ (b) $\sqrt{\frac{4}{5}}$
(c) $\sqrt{\frac{3}{5}}$ (d) $\sqrt{\frac{14}{15}}$

5. **(d)**

$$v = \sqrt{\frac{2gH}{1 + k^2/R^2}}$$

$$\frac{V_{\text{Cylinder}}}{V_{\text{Sphere}}} = \sqrt{\frac{\left(1 + k^2/R^2\right)_{\text{Sphere}}}{\left(1 + k^2/R^2\right)_{\text{Cylinder}}}}$$

$$= \sqrt{\frac{1 + 2/5}{1 + 1/2}} = \sqrt{\frac{14}{15}}$$

Q.6 Three identical particles A, B and C of mass 100 kg each are placed in a straight line with $AB = BC = 13$ m. The gravitational force on a fourth particle P of the same mass is F, when placed at a distance 13 m from the particle B on the perpendicular bisector of the line AC. The value of F will be approximately :

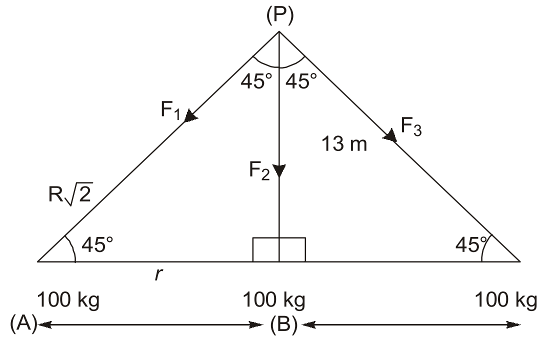
(a) 21 G (b) 100 G
(c) 59 G (d) 42 G

6. (b)

$$F \text{ on } P = \frac{GMM}{r^2} + \sqrt{2}\frac{GMM}{(\sqrt{2}r)^2}$$

$$= \frac{GMM}{r^2}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$F \text{ on } P = \frac{G\times 10^4}{13^2}\left(1+\frac{1}{\sqrt{2}}\right) \approx 100\,G$$



Q.7 A certain amount of gas of volume V at 27°C temperature and pressure 2×10^7 Nm^{-2} expands isothermally until its volume gets doubled. Later it expands adiabatically until its volume gets redoubled. The final pressure of the gas will be

(Use $\gamma = 1.5$) :

(a) 3.536×10^5 Pa
(b) 3.536×10^6 Pa
(c) 1.25×10^6 Pa
(d) 1.25×10^5 Pa

7. (b)

$$P_1 = 2\times 10^7\ Pa$$

$$P_1 v_1 = P_2 v_2$$

Since $v_2 = 2v_1$ Hence $P_2 = \frac{P_1}{2}$ (Isothermal Expansion)

$$P_2 = 1\times 10^7\ Pa$$

$$P_2 (v_2)^y = P_3 (2v_2)^y$$

$$P_3 = \frac{1\times 10^7}{2^{1.5}} = 3.536\times 10^6 Pa$$

Q.8 Following statements are given :

1. The average kinetic energy of a gas molecule decreases when the temperature is reduced.
2. The average kinetic energy of a gas molecule increases with increase in pressure at constant temperature.
3. The average kinetic energy of a gas molecule decreases with increase in volume.
4. Pressure of a gas increases with increase in temperature at constant pressure.
5. The volume of gas decreases with increase in temperature.

Choose the correct answer from the options given below :

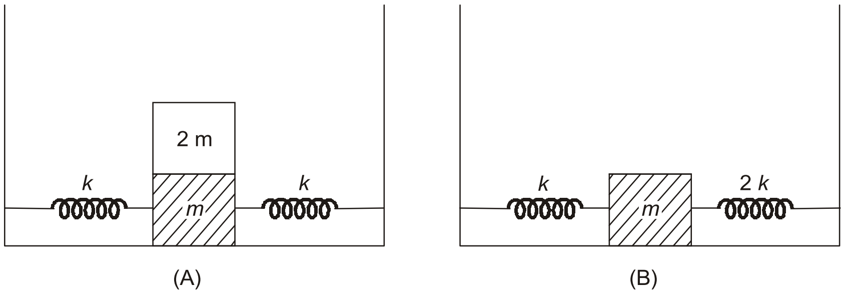
(a) 1 and 4 only
(b) 1, 2 and 4 only
(c) 2 and 4 only
(d) 1, 2 and 5 only

8. (a)

$$KE_{avg} = \frac{3}{2}KT$$

$$P = \frac{1}{3}\rho V_{rms}^2$$

Q.9 In figure (A), mass '2 m' is fixed on mass 'm' which is attached to two spring of spring constant k. In figure (B), mass 'm' is attached to two springs of spring constant 'k' and '$2k$' .If mass 'm' in (A) and in (B) are displaced by distance 'x' horizontally and then released, then time period T_1 and T_2 corresponding to (A) and (B) respectively follow the relation.



(a) $\frac{T_1}{T_2} = \frac{3}{\sqrt{2}}$

(b) $\frac{T_1}{T_2} = \sqrt{\frac{3}{2}}$

(c) $\frac{T_1}{T_2} = \sqrt{\frac{2}{3}}$

(d) $\frac{T_1}{T_2} = \frac{\sqrt{2}}{3}$

9. (a)

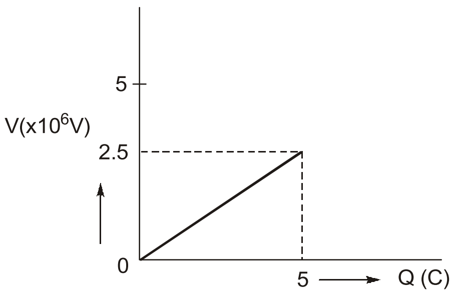
$$T_1 = 2\pi\sqrt{\frac{3m}{2k}}$$

$$T_2 = 2\pi\sqrt{\frac{m}{3k}}$$

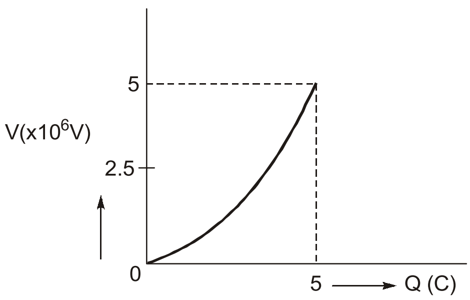
$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{3m}{2k}}}{2\pi\sqrt{\frac{m}{3k}}} = \frac{3}{\sqrt{2}}$$

Q.10 A condenser of 2μF capacitance is charged steadily from 0 to 5 C. Which of the following graph represents correctly the variation of potential difference (V) across it's plates with respect to the charge (*Q*) on the condenser?

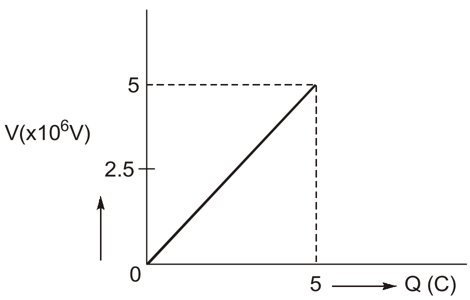
(a)



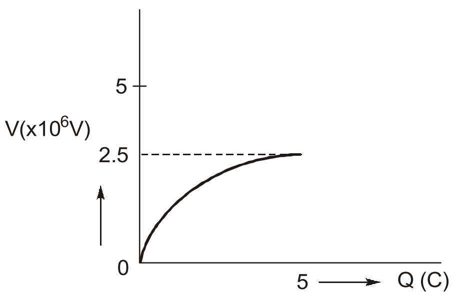
(b)



(c)



(d)



10. (a)

$$Q = CV$$

$$V = \frac{Q}{C}$$

$$\text{Straight line with slope} = \frac{1}{C}$$

$$\text{Slope} = \frac{1}{C} = \frac{1}{2\times10^{-6}} = 5\times10^{5}$$

Q.11 Two charged particles, having same kinetic energy, are allowed to pass through a uniform magnetic field perpendicular to the direction of motion. If the ratio of radii of their circular paths is 6 : 5 and their respective masses ratio is 9 : 4. Then, the ratio of their charges will be :

(a) 8 : 5
(b) 5 : 4
(c) 5 : 3
(d) 8 : 7

11. (b)

$$\text{Radius of circular path } R = \sqrt{\frac{2mk}{qB}}$$

$$q = \frac{\sqrt{2mk}}{RB}$$

$$\frac{q_1}{q_2} = \sqrt{\frac{m_1}{m_2}}\times\frac{R_2}{R_1} = \sqrt{\frac{9}{4}}\times\frac{5}{6} = \frac{5}{4}$$

Q.12 To increase the resonant frequency in series LCR circuit,

(a) Source frequency should be increased.
(b) Another resistance should be added in series with the first resistance.
(c) Another capacitor should be added in series with the first capacitor.
(d) The source frequency should be decreased.

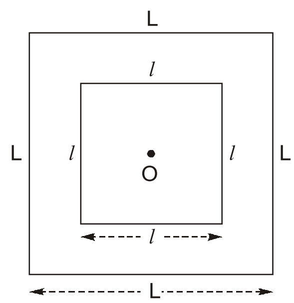
12. (c)

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$f \propto \frac{1}{\sqrt{C}}$$

Another capacitor should be added in series with the first capacitor.

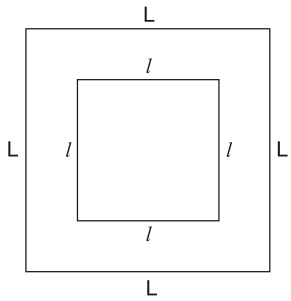
Q.13 A small square loop of wire of side l is placed inside a large square loop of wire. $L(L>>l)$. Both loops are coplanar and their centres coincide at point O as shown in figure. The mutual inductance of the system is:



(a) $\frac{2\sqrt{2}\mu_0 L^2}{\pi l}$
(b) $\frac{\mu_0 l^2}{2\sqrt{2}\pi L}$
(c) $\frac{2\sqrt{2}\mu_0 l^2}{\pi L}$
(d) $\frac{\mu_0 L^2}{2\sqrt{2}\pi l}$

13. (c)

Assuming current 'i' In outer loop magnetic field at centre



$$= 4 \times \frac{\mu_0}{4\pi}\frac{i}{L/2} \times \left[2\sin 45^\circ\right)$$

$$= \frac{2\sqrt{2}\mu_0 i}{\pi L}$$

$$M = \frac{\text{F m } \alpha \text{ through inner loop}}{i}$$

$$= \frac{2\sqrt{2}\mu_0 i l^2}{\pi L} = \frac{2\sqrt{2}\mu_0 l^2}{\pi L}$$

Q.14 The rms value of conduction current in a parallel plate capacitor is 6.9 μA. The capacity of this capacitor, if it is connected to 203 V ac supply with an angular frequency of 600 rad / s, will be:

(a) 5 pF (b) 50 pF
(c) 100 pF (d) 200 pF

14. (b)

$$\text{Current in capacitor } I = \frac{V}{X_c}$$

$$I = V \times (\omega C)$$

$$C = \frac{I}{V\omega} = \frac{6.9 \times 10^{-6}}{230 \times 600} = 50 \text{ pF}$$

Q.15 Which of the following statement is correct ?

(a) In primary rainbow, observer sees red colour on the top and violet on the bottom
(b) In primary rainbow, observer sees violet colour on the top and red on the bottom
(c) In primary rainbow, light wave suffers total internal reflection twice before coming out of water drops.
(d) Primary rainbow is less bright than secondary rainbow.

15. (a)

In primary rainbow, red colour is at top and violet is at bottom.
Intensity of secondary rainbow is less in comparison to primary rainbow.

Q.16 Time taken by light to travel in two different materials A and B refractive indices μ_A and μ_B of same thickness is t_1 and t_2 respectively. If $t_2 - t_1 = 5 \times 10^{-10}$s and the ratio of μ_A to μ_B is 1 : 2. Then, the thickness of material, in material, in meter is : (Given υ_A and υ_B are velocities of light in A and B materials respectively)

(a) $5 \times 10^{-10}\, \upsilon_A$ m (b) 5×10^{-10} m
(c) 1.5×10^{-10} m (d) $5 \times 10^{-10}\, \upsilon_B$ m

16. (a)

$$\frac{\mu_A}{\mu_B} = \frac{C/v_A}{C/v_B} = \frac{v_B}{v_A} = \frac{1}{2}$$

Let the thickness is d

$$\frac{d}{v_B} - \frac{d}{v_A} = 5 \times 10^{-10}$$

$$d = \frac{5 \times 10^{-10} \times v_A v_B}{v_A - v_B}$$

$$\text{As } V_A = 2v_B \Rightarrow d = 5 \times 10^{-10} \times 2v_B$$

$$\text{or, } d = 5 \times 10^{-10} \times v_A$$

Q.17 A metal exposed to light of wavelength 800 nm and and emits photoelectrons with a certain kinetic energy. The maximum kinetic energy of photo-electron doubles when light of wavelength 500 nm is used. The work function of the metal is : (Take hc = 1230 eV -nm).

(a) 1.537 eV (b) 2.46 eV

(c) 0.615 eV (d) 1.23 eV

17. (c)

$$K.E_1 = \frac{1230}{800} - \phi \quad ...(i)$$

$$K.E_2 = 2K \cdot E_1 = \frac{1230}{500} - \phi \quad ...(ii)$$

From (*i*) and (*ii*)

$$0 = \frac{1230}{500} - \frac{1230}{400} + \phi$$

$$\phi = 0.615\ ev$$

Q.18 The momentum of an electron revolving in n^{th} orbit is given by :
(Symbols have their usual meanings)

(a) $\frac{nh}{2\pi r}$ (b) $\frac{nh}{2r}$

(c) $\frac{nh}{2\pi}$ (d) $\frac{2\pi r}{nh}$

18. (a)

$$mvr = \frac{nh}{2\pi}$$

$$\text{So momentum } (mv) = \frac{nh}{2\pi r}$$

Q.19 The magnetic moment of an electron (*e*) revolving in an orbit around nucleus with an orbital angular momentum is given by :

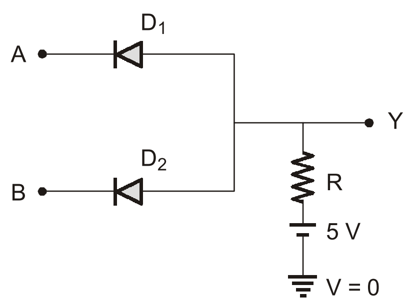
(a) $\vec{\mu}_L = \frac{e\vec{L}}{2m}$ (b) $\vec{\mu}_L = -\frac{e\vec{L}}{m}$

(c) $\vec{\mu}_l = -\frac{e\vec{L}}{m}$ (d) $\vec{\mu}_l = \frac{2e\vec{L}}{m}$

19. (b)

$$\vec{\mu} = -\frac{e}{2m}\vec{L}$$

Q.20 In the circuit, the logical value of $A = 1$ or $B = 1$ when potential at A or B is 5 V and the logical value of $A = 0$ or $B = 0$ when potential at A or B is 0 V. The truth table of the given circuit will be :



(a)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

(b)

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

(c)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	0

(d)

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

20. (a)

When both A and B have logical value 'I' both diode are revere bias and current will flow in resistor hence output will be 5 volt i.e logical value '1'.

In all other case conduction will take place hence output will be zero value i.e logical value 'o'.

So truth table is

A	B	Y	
0	0	0	
0	1	0	(AND gate)
1	0	0	
1	1	1	

SECTION - B

Q.1 A car is moving with speed of 150 km/h and after applying the break it will move 27 m before it stops. If the same car is moving with a speed of one third the reported speed then it will stop after travelling __________ m distance.

1. (3)

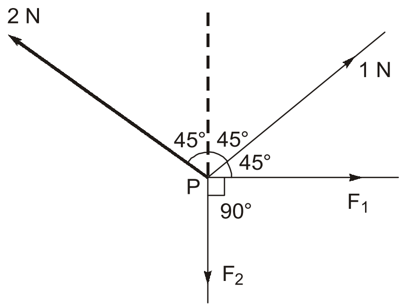
$$\text{Stopping distance } \frac{v^2}{2a} = d$$

$$\text{If speed is made } \frac{1}{3}\text{rd}$$

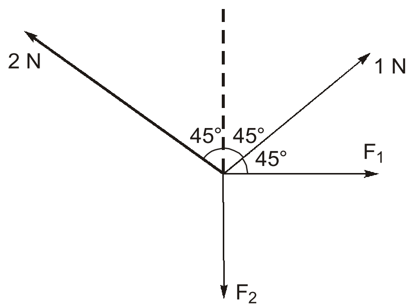
$$d^1 = \frac{d}{9} \quad , \quad d^1 = \frac{27}{9} = 3 \text{ m}$$

Breaking Acceleration Remains same.

Q.2 Four forces are acting at a point P in equilibrium as shown in figure. The ratio of force F_1 to F_2 is 1 : x where x = __________.



2. (3)



$$F_2 = 1 \cos 45° + 2 \cos 45° = 3 \cos 45° = \frac{3}{\sqrt{2}} N$$

$$F_1 + 1 \cos 45° = 2 \sin 45°$$

$$F_1 + 1 \sin 45° = \frac{1}{\sqrt{2}} N$$

$$\frac{F_1}{F_2} = 1 : 3$$

$$x = 3$$

Q.3 A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force F, its length increases by 5 cm. Another wire of the same material of length 4 L and radius 4 r pulled by a force 4 F under same conditions. The increase in length of this wire is ________ cm.

3. (5)

$$\Delta L_1 = \frac{FL}{AY} = \frac{FL}{\pi r^2 Y} = 5 \text{ cm}$$

$$\Delta L_2 = \frac{4F4L}{\pi 16 r^2 Y} = \frac{FL}{\pi r^2 Y} = 5 \text{ cm}$$

Q.4 A unit scale is to be prepared whose length does not change with temperature and remains 20 cm, using a bimetallic strip made of bass and iron each of different length. The length of both components would change in such a way that difference between their length remains constant. If length of brass is 40 cm and length of iron will be ________ cm.

($\alpha_{iron} = 1.2 \times 10^{-5}\ K^{-1}$ and $\alpha_{brass} = 1.8 \times 10^{-5}\ K^{-1}$).

4. (60)

$$l_B(1 + \alpha_B \Delta T) - l_i(1 + \alpha_i \Delta T) = l_B - l_i$$

$$\Rightarrow \quad \alpha_B l_B = l_i \alpha_i$$

$$\Rightarrow \quad 1.8 \times 10^{-5} \times 40 = l_i \times 1.2 \times 10^{-5}$$

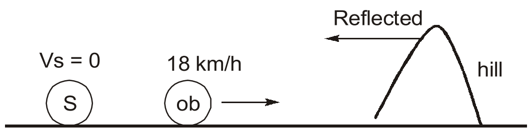
$$\Rightarrow \quad l_i = \frac{1.8 \times 10^{-5} \times 40}{1.2 \times 10^{-5}} = \frac{3 \times 40}{2} = 60$$

$$l_i = 60 \text{ cm}$$

Q.5 An observer is riding on a bicycle and moving towards a hill at 18 kmh^{-1}. He hears a sound from a source at some distance behind him directly as well as after its reflection from the hill. If the original frequency of the sound as emitted by source is 640 Hz and velocity of the sound in air is 320 m/s, the beat frequency between the two sounds heard by observer will be _______ Hz.

5. (20)

$$V_S = 0, \ V_{O_b} = 5 \text{ m/s}$$



$$f_{direct} = \left(\frac{320 - 5}{320}\right) 640 = 630 \text{ Hz}$$

$$f_{reflected} = \left(\frac{320 + 5}{320}\right) 640 = 650 \text{ Hz}$$

$$f_{beat} = (650 - 630) = 20 \text{ Hz}$$

Q.6 The volume charge density of a sphere of radius 6 m is 2 $\mu C\ cm^{-3}$. The number of lines of force per unit surface area coming out from the surface of the sphere is _________ $\times 10^{10}\ NC^{-1}$.

[Given : Permittivity of vacuum $\epsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1} - m^{-2}$]

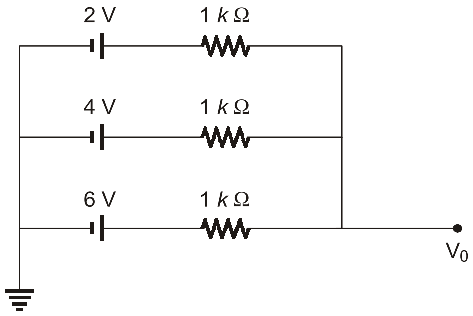
6. (45)

$$E = \frac{\rho r}{3 \epsilon_0} \text{ for } r = R$$

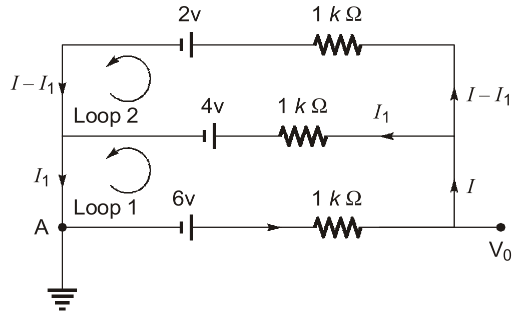
$$E = \frac{\rho R}{3 \epsilon_0} = \frac{2 \times 6}{3 \times 8.85 \times 10^{-12}} = 0.45 \times 10^{12}\ NC^{-1}$$

$$E = 45 \times 10^{10} \text{ N/C}$$

Q.7 In the given figure, the value of V_0 will be _______ V.



7. (4)



K V L Loop 1

$$6 - I \times 1 - I_1 \times 1 - 4 = 0$$

$$2 = I + I_1 \quad \ldots(i)$$

K V L Loop 2

$$4 + I_1 \times 1 - (I - I_1) \times 1 - 2 = 0$$

$$\Rightarrow \quad 2 + I_1 - I = 0$$

$$2 + 2I_1 - I = 0$$

$$I = 2 + 2I_1 \quad \ldots(ii)$$

from (i) and (ii)

$$2 = 2 + 2I_1 + I_1 \qquad I = 2\text{A}$$

$$3I_1 = 0 \qquad V_A + 6 - I \times 1 = v_0$$

$$\Rightarrow 0 + 6 - 2 = v_0$$

$$I_1 = 0 \qquad v_0 = 4$$

Q.8 Eight copper wire of length l and diameter d are joined in parallel to from a single composite conductor of resistance R. If a single copper wire of length 2 l have the same resistance (R) then its diameter will be ________ d.

8. (4)

Each wile has resistance $= \dfrac{\rho 4\ell}{\pi d^2} = r$

Eight wire in parallel, then equivalent resistance is

$$\frac{r}{8} = \frac{\rho l}{2\pi d^2}$$

Single copper wire of length $2l$ has resistance

$$R = \rho \frac{2l \times 4}{\pi d_1^2} = \frac{\rho l}{2\pi d^2}$$

$$d_1 = 4\,d$$

Q.9 The energy band gap of semiconducting material to produce violet (wavelength 4000Å). LED is _______ eV. (Round off to the nearest integer).

9. (3)

$$E_g = \frac{hc}{\lambda} = \frac{1242}{\lambda(\text{nm})} = \frac{1242}{400} = 3.105$$

Q.10 The required height of a TV tower which can cover the population of 6.03 lakh is h. If the average population density is 100 per square km and the radius of earth is 6400 km, then the value of h will be _________ m.

10. (150)

$$d = \sqrt{2Rh}$$

$$d = \sqrt{2 \times 6400 \times h \times 10^{-3}} \quad (h \text{ in } m)$$

$$\text{Area} = \pi d^2$$

$$= (\pi \times 2 \times 6400 \times h \times 10^{-3})\ \text{km}^2$$

$$6.03 \times 100000 = 100 \times \pi \times 2 \times 6400 \times 10^{-3}\ h$$

$$h = \frac{6.03 \times 10^5}{10 \times \pi \times 128}$$

$$h = 150\ m$$

PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q.1 SO_2Cl_2 on reaction with excess of water results into acidic mixture

$SO_2Cl_2 + 2H_2O \rightarrow H_2SO_4 + 2HCl$

16 moles of NaOH is required for the complete neutralisation of the resultant acidic mixture. The number of moles of SO_2Cl_2 used is:

(a) 16 (b) 8
(c) 4 (d) 2

1. **(c)**

$SO_2Cl_2 + 2H_2O \rightarrow H_2SO_4 + 2HCl$

Let a moles of SO_2Cl_2 is taken

Then no. of moles of H_2SO_4 = a moles

No. of moles of HCl = $2a$ moles

No. of moles of NaOH required = $2a + 2a = 4a = 16$

$\Rightarrow \quad a = 4$ moles

Q.2 Which of the following sets of quantum numbers is not allowed?

(a) $n = 3,\ l = 2,\ m_1 = 0,\ s = \frac{1}{2}$ (b) $n = 3,\ l = 2,\ m_1 = 2,\ s = +\frac{1}{2}$

(c) $n = 3,\ l = 3,\ m_1 = -3,\ s = -\frac{1}{2}$ (d) $n = 3,\ l = 0,\ m_1 = 0,\ s = -\frac{1}{2}$

2. **(c)**

Value of n and l can't be same

Q.3 The depression in freezing point observed for a formic acid solution of concentration 0.5 mL L^{-1} is 0.0405°C. Density of formic acid is 1.05 g mL^{-1}. The Van't Hoff factor of the formic acid solution is nearly: (Given for water K_f = 1.86 k kg mol^{-1}).

(a) 0.8 (b) 1.1
(c) 1.9 (d) 2.4

3. **(c)**

Let we take 1 l of solution

$$\begin{aligned}
\text{Mass of solute} &= \text{Volume} \times \text{Density} \\
&= 0.5\ \text{ml} \times 1.05\ \text{gm/ml} \\
&= 0.525\ \text{gram} \\
\text{Mass of solution} &= 1\ \text{kg. [considering very dilute solution]} \\
\text{Mass of solvent} &= 1000 - 0.525 = 999.475\ \text{gram} \\
\Delta T &= i \times K_t \times M
\end{aligned}$$

$$\Rightarrow \quad 0.0405 = i \times 1.86 \times \left(\frac{0.525 \times 1000}{46 \times 999.475}\right)$$

$$\Rightarrow \quad i = 1.9$$

Q.4 20 mL of 0.1 M NH_4OH is mixed with 40 mL of 0.05 M HCl. The pH of the mixture is nearest to:

(Given K_b (NH_4OH) = 1×10^{-5}, log 2 = 0.30, log 3 = 0.48, log 5 = 0.69, log 7 = 0.84, log 11 = 1.04

(a) 3.2 (b) 4.2

(c) 5.2 (d) 6.2

4. (c)

$$[NH_4Cl] = \frac{2}{60} = \frac{1}{30} M$$

$$pH = 7 - \frac{1}{2} PK_b - \frac{1}{2} \log C$$

$$= 7 - \frac{5}{2} - \frac{1}{2} \log\left(\frac{1}{30}\right) = 5.24$$

Q.5 Match the List-I with List-II

	List-I		List-II
A.	$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$	**1.**	Cu
B.	$CO(g) + 3H_2(g) \rightarrow CH_4(g) + H_2O(g)$	**2.**	$Cu/ZnO - Cr_2O_3$
C.	$CO(g) + H_2(g) \rightarrow HCOH(g)$	**3.**	$Fe_xO_y + K_2O + Al_2O_3$
D.	$CO(g) + 2H_2(g) \rightarrow CH_3OH(g)$	**4.**	Ni

Choose the correct answer from the options given below:

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	2	1	4	3
(c)	3	4	1	2
(d)	3	1	4	2

5. (c)

Uses of catalyst is very specific for a particular reaction.

Q.6 The IUPAC nomenclature of an element with electronic configuration [Rn] $5f^{14}6d^17s^2$ is:

(a) Unnilbium (b) Unnilunium

(c) Unnilquandium (d) Unniltrium

6. (d)

IUPAC nomenclature of element with atomic no. 103 is uniltrium.

Q.7 The compound(s) that is (are) removed as slag during the extraction of copper is:

1. CaO
2. FeO
3. Al_2O_3
4. ZnO
5. NiO

Choose the correct answer from the options given below:

(a) 3 and 4 only (b) 1, 2 and 5 only

(c) 1 and 2 only (d) 2 only

7. **(d)**

$\underset{\text{(Acidic flux)}}{FeO + SiO_2} \longrightarrow \underset{\text{(Slag)}}{FeSiO_3}$

Q.8 The reaction of H_2O_2 with potassium permanganate in acidic medium leads to the formation of mainly.
(a) Mn^{2+} (b) Mn^{4+}
(c) Mn^{3+} (d) Mn^{6+}

8. **(a)**

$$2MnO_4^- + 6H^+ + 5H_2O_2 \longrightarrow 2Mn^{2+} + 8H_2O + 5O_2$$

Q.9 Choose the **correct** order of density of the alkali metals:
(a) Li < K < Na < Rb < Cs (b) Li < Na < K < Rb < Cs
(c) Cs < Rb < K < Na < Li (d) Li < Na < K < Cs < Rb

9. **(a)**
Density of metal a very specific and depends upon many factors.

$Li \longrightarrow 0.53\ gm / cm^3$

$Na \longrightarrow 0.97\ gm / cm^3$

$K \longrightarrow 0.86\ gm / cm^3$

$Rb \longrightarrow 1.53\ gm / cm^3$

$Cs \longrightarrow 1.90\ gm / cm^3$

Q.10 The geometry around boron in the product 'B' formed from the following reaction is

$BF_3 + NaH \xrightarrow{450\ K} A + NaF$

$A + NMe_3 \longrightarrow B$

(a) Trigonal planar (b) Tetrahedral
(c) Pyramidal (d) Square planar

10. **(b)**

$2BF_3 + 6\ NaH \longrightarrow B_2H_6 + 6\ NaF$

$B_2H_6 + 2\ \ddot{N}(Me)_3 \longrightarrow 2H_3B \leftarrow :N(Me)_3$

Q.11 The interhalogen compound formed from the reaction of bromine with excess of fluorine is:
(a) Hypohalite (b) Halate
(c) Perhalate (d) Halite

11. **(b)**

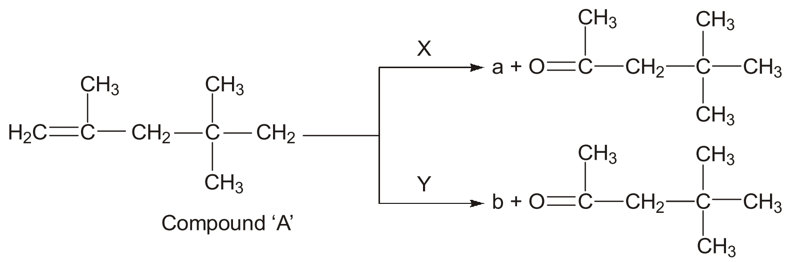
$Br_2 + 5F_2\ (excess) \longrightarrow 2BrF_5$

$BrF_5 + 3H_2O \longrightarrow HBrO_3 + 5HF$

Q.12 The photochemical smog does not generally contain
(a) NO (b) NO_2
(c) SO_2 (d) HCHO

12. **(c)**
Photochemical smog generally does not content SO_2 gas.

Q.13 A compound 'A' on reaction '*X*' and '*Y*' produces the same major product but different by product '*a*' and '*b*'. Oxidation of '*a*' gives a substance produced by ants.



'*X*' and '*Y*' respectively are

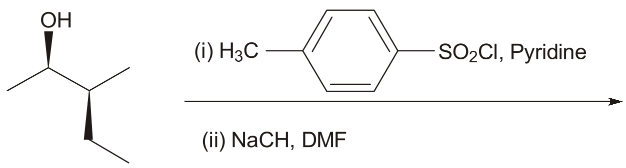
(a) $KMnO_4$ / H^+ and dil. $KMnO_4$, 273 K (b) $KMnO_4$ (dilute), 273 K and $KMnO_4$ / H^+

(c) $KMnO_4$ / H^+ and O_3, H_2O / Zn (d) O_3, H_2O / Zn and $KMnO_4$ / H^+

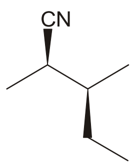
13. (d)

Oxidation of HCHO(a) will produce HCOOH, which is produced by ants.

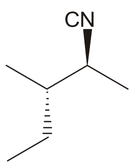
Q.14 Most stable product of the following reaction is:



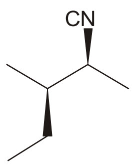
(a)



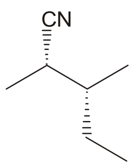
(b)



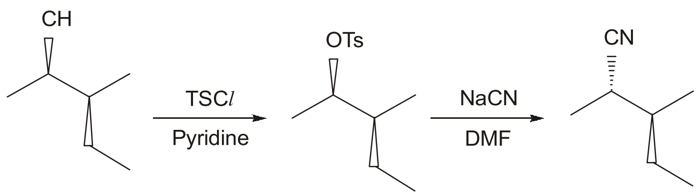
(c)



(d)

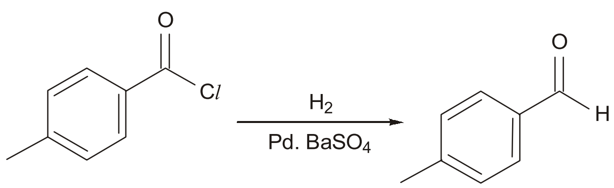


14. (b)

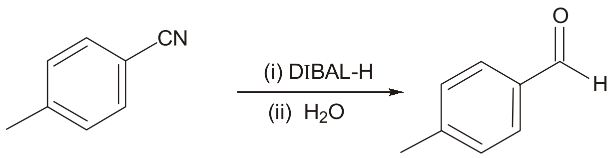


Q.15 Which one of the following reactions does not represent correct combination of substrate and product under the given condition?

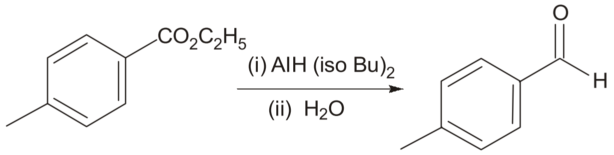
(a)



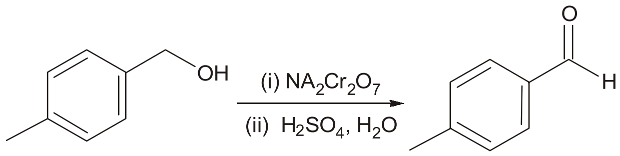
(b)



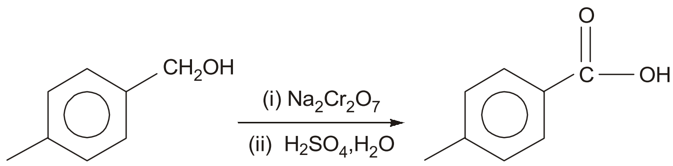
(c)



(d)

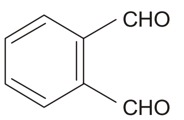


15. **(d)**

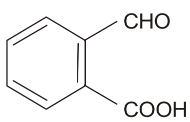


Q.16 An organic compound '*A*' on reaction with NH_3 followed by heating gives compound *B*. Which on further strong heating gives compound $C(C_8H_5NO_2)$. Compound *C* on sequential reaction with ethanolic KOH, alkyl chloride and hydrolysis with alkali gives a primary amine. The compound *A* is:

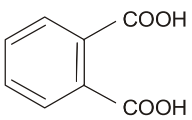
(a)



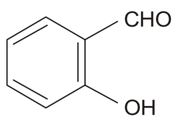
(b)



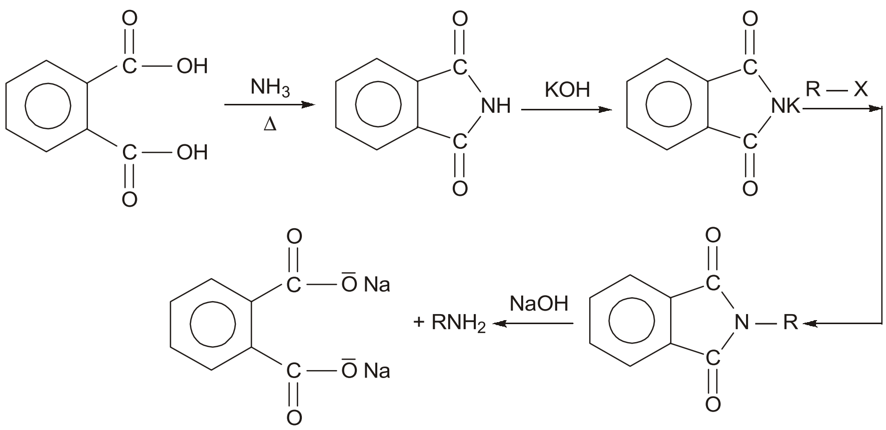
(c)



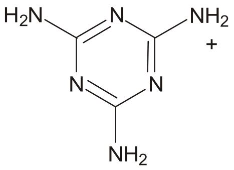
(d)

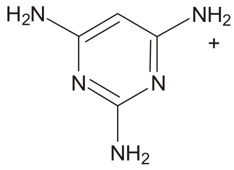


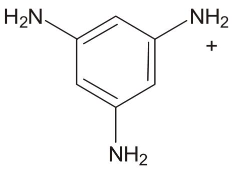
16. **(c)**

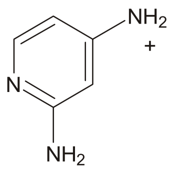


Q.17 Melamine polymer is formed by the condensation of:

(a) 

(b) 

(c) 

(d) 

17. **(a)**

Melamine + formaldehyde $\xrightarrow{\Delta}$ Melamine formaldehyde polymer (Malamac)

Q.18 During the denaturation of proteins, which of these structure will remain intact?

(a) Primary
(b) Secondary
(c) Tertiary
(d) Quaternary

18. **(a)**

During denaturation of protein, 1° structure remains the same, whereas 2° and 3° structure get destroyed.

Q.19 Drugs used to bind to receptors inhibiting its natural function and blocking a message are called :

(a) Agonists
(b) Antagonists
(c) Allosterists
(d) Anti histaminists

19. **(b)**

Based upon properties of drugs.

Q.20 Given below are two statements:

Statement I: On heating with $KHSO_4$, glycerol is dehydrated and acrolein is formed.

Statement II: Acrolein has fruity odour and can be used to test glycerol's presence.

Choose the correct option.

(a) Both **Statement I** and **Statement II** are correct.
(b) Both **Statement I** and **Statement II** are incorrect.
(c) **Statement I** is correct but **Statement II** is incorrect.
(d) **Statement I** is incorrect but **Statement II** is correct.

20. **(c)**

Glycerol $\xrightarrow[\Delta]{KHSO_4}$ Acrolein (Pungest, suffocating odour)

SECTION - B

Q.1 Among the following species

$N_2, N_2^+, N_2^-, N_2^{2-}, O_2, O_2^+, O_2^-, O_2^{2-}$

The number of species showing diamagnesim is ________________.

1. **(2)**

In N_2 and $O_2^{2-} \rightarrow$ no. of unpaired electron = 0

In other species $\rightarrow$ no. of unpaired electron $\neq 0$

Q.2 The enthalpy of combustion of propane, graphite and dihydrogen at 298 K are -2220.0 kJ mol^{-1}, – 393.5 kJ mol^{-1} and – 285.8 kJ mol^{-1} respectively. The magnitude of the enthalpy of formation of propane C_3H_8 is __________ kJ mol^{-1} (Nearest integer)

2. **(104)**

$3C(\text{Graphite}) + 4H_2(g) \longrightarrow C_3H_8(g)$

$$\Delta H_f(C_3H_8) = [3\times\Delta H_{comb}(C)] + [4\times\Delta H_{comb}(H_2)] - [\Delta H_{comb}(C_3H_8)]$$
$$= 10.3.7 \text{ kJ / mole}$$

Q.3 The pressure of a moist gas at 27°C is 4 atm. The volume of the container is doubled at the same temperature. The new pressure of the moist gas is____________ $\times 10^{-1}$ atm. (Nearest integer)

(Given: The vapour pressure of water at 27°C is 0.4 atm)

3. **(22)**

$$\text{Pressure of gas} = \text{Pressure of moist gas} - \text{Vapour pressure of water}$$
$$= 4 \text{ atm} - 0.4 \text{ atm}$$
$$= 3.6 \text{ atm}$$

On doubling the volume, pressure of gas is halved:

$$\text{Total pressure} = \left(\frac{3.6}{2} + 0.4\right) = 2.2 \text{ atm}$$

Q.4 The cell potential for Zn| Zn^{2+} (aq) || Sn^{x+} | Sn is 0.801 V at 298 K. The reaction quotient of the above reaction is 10^{-2}. The number of electrons involved in the given electrochemical cell reaction is__________________.

(Given: $E^{\circ}_{Zn^{2+}|Zn} = -0.763\text{V}, E^{\circ}_{Sn^{x+}|Sn} = +0.008\text{V}$ and $\frac{2.303\,RT}{F} = 0.06\text{V}$)

4. **(4)**

Anode : $(Zn \longrightarrow Zn^{2+} + 2e)\times x$

Cathode : $(Sn^{x+} + xe \longrightarrow Sn)\times 2$

Net cell reaction : $xZn + 2Sn^{x+} \longrightarrow xZn^{2+} + 2Sn\ (n = 2x)$

Using Nernst equation

$$E_{cell} = E^0_{cell} - \frac{0.06}{2x}\log Q$$

$$\Rightarrow \quad 0.801 = 0.771 - \frac{0.06}{2x}\log 10^{-2}$$

$$\Rightarrow \quad x = 2$$

No. of electron involved = 4

Q.5 The half life for the decomposition of gaseous compound A is 240s when the gaseous pressure was 500 Torr initially, When the pressure was 250 Torr, the half life was found to be 4.0 min. The order of the reaction is ___________. (Nearest integer)

5. **(1)**

$$\frac{(t_{1/2})_I}{(t_{1/2})_{II}} = \left(\frac{P_I}{P_{II}}\right)^{1-n} \Rightarrow n = 1$$

Q.6 Consider the following metal complexes:

$[Co(NH_3)_6]^{3+}$

$[CoCl(NH_3)_5]^{2+}$

$[Co(CN)_6]^{3-}$

$[Co(NH_3)_5(H_2O)]^{3+}$

The spin-only magnetic moment value of the complex that absorbes light with shortest wavelength is ________________ B.M (Nearest integer)

6. **(0)**

Complex absorbing minimum wavelength of light means the value of Δ for it is maximum.

That complex will be $[Co(CN)_6]^{3-}$

$CO^3 + \Rightarrow 3d^6\,4s^{\circ} \Rightarrow t_2g^6 eg^{\circ}$

No. of unpaired electron = 0

Q.7 Among Co^{3+}, Ti^{2+}, V^{2+} and Cr^{2+} ions, one if used as a reagent cannot liberate H_2 from dilute mineral acid solution, its spin-only magnetic moment in gaseous state is __________B.M. (Nearest integer)

7. **(5)**

Co^{3+} will not get oxidized, so will not liberate H_2 gas upon reaction with acidic solution.

$Co^{3+} \Rightarrow 3d^6 4s^0$ | ⥮ | ↿ | ↿ | ↿ | ↿ |

No. of unpaired electron = 4

Spin only magnetic moment = $\sqrt{4(4+2)}\ \text{BM} = \sqrt{24}\ \text{BM} \approx 5\ \text{BM}$

Q.8 While estimating the nitrogen present in an organic compound by Kjeldahl's method, the ammonia evolved from 0.25 g of the compound neutralized 2.5 mL of M H_2SO_4. The percentage of nitrogen present in organic compound is__________.

8. **(56)**

Organic compound $\longrightarrow NH_3$

$2NH_3 + H_2SO_4 \longrightarrow (NH_4)_2SO_4$

$$\begin{aligned}\text{No. of milimoles of } NH_3 &= 2\text{x no. of milimoles of } H_2SO_4\\ &= 2\times 2\times 2.5\\ &= 10 \text{ milimoles}\\ &= \text{No. of milimoles of N}\end{aligned}$$

$$\text{Mass of nitrogen} = \frac{10}{1000}\times 14 = 0.14 \text{ gm}$$

$$\%\text{ of Nigrogen in compound} = \frac{0.14}{0.25}\times 100 = 56\%$$

Q.9 The number of sp^3 hybridised carbons in an acyclic neutral compound with molecular formula C_4H_5N is________________.

9. **(1)**

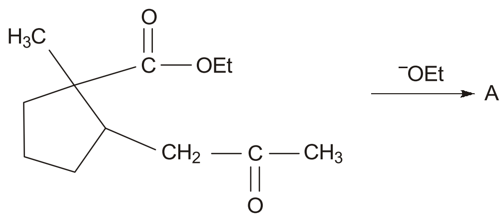
$$\text{Degree ofunsaturation} = 4+1-\left(\frac{5-1}{2}\right) = 3$$

$Ch_3—CH{=}CH-C{\equiv}N$

↑

sp^3 hybridized only one carbon

Q.10 In the given reaction



(Where Et is $-C_2H_5$)

The number of chiral carbon / s in product *A* is____________.

10. **(2)**

2 (only 2 carbon will chiral in product form)

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q.1 The total number of functions, $f : \{1, 2, 3, 4\} \to \{1, 2, 3, 4, 5, 6\}$ such that $f(1) + f(2) = f(3)$, is equal to :

(a) 60 (b) 90

(c) 108 (d) 126

1. (b)

Case 1 : If $f(3) = 3$ then $f(1)$ and $f(2)$ take 1 OR 2

No. of ways = 2.6 = 12

Case 2 : If $f(3) = 5$ then $f(1)$ and $f(2)$ take 2 OR 3

OR 1 and 4

No. of ways = 2.6.2 = 24

Similarly for all other cases.

Total no. of ways 90.

Q.2 If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to :

(a) –4 (b) –1

(c) 1 (d) 4

2. (b)

From properties of nth root of unity

$1^{2021} + \alpha^{2021} + \beta^{2021} + \Upsilon^{2021} + \delta^{2021} = 0$

$\Rightarrow \alpha^{2021} + \beta^{2021} + \Upsilon^{2021} + \delta^{2021} = -1$

Q.3 For $n \in N$, let $S_n = \left\{z \in C : |z - 3 + 2i| = \frac{n}{4}\right\}$ and $T_n = \left\{z \in C : |z - 2 + 3i| = \frac{1}{n}\right\}$. Then the number of elements in the set $\{n \in N : S_n \cap T_n = \phi\}$ is :

(a) 0 (b) 2

(c) 3 (d) 4

3. (DROP)

$S_n = \left\{z \in C : |z - 3 + 2i| = \frac{n}{4}\right\}$

represents a circle with centre C_1 (3, – 2) and radius

$$r_1 = \frac{n}{4}$$

Similarly T_n represent circle with centre C_2 (2, – 3) and radius

$$r_1 = \frac{1}{n}$$

$$\text{As } S_n \cap T_n = \phi$$

$$C_1C_2 > r_1 + r_2 \text{ OR } C_1C_2 < |r_1 - r_2|$$

$$\sqrt{2} > \frac{n}{4}+\frac{1}{n}$$

OR

$$\sqrt{2} < \left|\frac{n}{4}+\frac{1}{n}\right|$$

n take infinite values.

Q.4 The number of $\theta \in (0, 4\pi)$ for which the system of linear equations

$3(\sin 3\theta)\, x - y + z = 2$

$3(\cos 2\theta)\, x + 4y + 3z = 3$

$6x + 7y + 7z = 9$, has no solution, is :

(a) 6 (b) 7

(c) 8 (d) 9

4. (b)

$$\Delta = \begin{vmatrix} 3\sin 3\theta & -1 & 1 \\ 3\cos 2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix}$$

$$= 3\sin 3\theta\,(28-21) + (21\cos 2\theta - 18) + 1\,(21\cos 2\theta - 24)$$

$$\Delta = 21\sin 3\theta + 42\cos 2\theta - 42$$

For no solution

$$\sin 3\theta + 2\cos 2\theta = 2$$

$$\Rightarrow \quad \sin 3\theta - 4\sin^3\theta = 4\sin^2\theta$$

$$\Rightarrow \quad \sin\theta\,(3 - 4\sin\theta - 4\sin^2\theta) = 0$$

$$\sin\theta = 0 \text{ OR } \sin\theta = \frac{1}{2}$$

$$\theta = \pi, 2\pi, 3\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

Q.5 If $\lim_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right) = 0$, the $8(\alpha+\beta)$ is equal to:

(a) 4 (b) – 8

(c) – 4 (d) 8

5. (c)

$$\lim_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right) = 0$$

$$\lim_{n\to\infty} n\left[\sqrt{1-\frac{1}{n}-\frac{1}{n^2}}+\alpha+\frac{\beta}{n}\right] = 0$$

$$\therefore \quad \alpha = -1$$

Now,

$$\lim_{n\to\infty} n\left[\left\{1-\left(\frac{1}{n}+\frac{1}{n^2}\right)\right\}^{\frac{1}{2}}+\frac{\beta}{n}-1\right] = 0$$

$$\lim_{n\to\infty}\frac{\left(1-\frac{1}{2}\left(\frac{1}{n}+\frac{1}{n^2}\right)+\ldots\right)+\frac{\beta}{n}-1}{\frac{1}{n}} = 0$$

$\Rightarrow \qquad \beta - \frac{1}{2} = 0$

$\therefore \qquad \beta = \frac{1}{2}$

Now, $\qquad 8(\alpha + \beta) = 8\left(-\frac{1}{2}\right) = -4$

Q.6 If the absolute maximum value of the function $f(x) = (x^2 - 2x + 7)\, e^{(4x^3 - 12x^2 + 180x + 31)}$ in the interval $[-3, 0]$ is $f(\alpha)$, then :

(a) $\alpha = 0$ (b) $\alpha = -3$

(c) $\alpha \in (-1, 0)$ (d) $\alpha \in (-3, -1)$

6. (b)

$$f(x) = \underbrace{(x^2 - 2x + 7)}_{f_1(x)} \underbrace{e^{(4x^3 - 12x^2 - 180x + 31)}}_{f_2(x)}$$

$$f_1(x) = x^2 - 2x + 7$$

$$f_1(x) = 2x - 2$$

So $f(x)$ is decreasing in $[-3, 0]$ and positive also

$$f_2(x) = e^{4x^3 - 12x^2 - 180x + 31}$$

$$f_2(x) = e^{4x^3 - 12x - 18x + 31} \cdot 12x^2 - 24x - 180$$

$$= 12(x - 5)(x + 3)\, e^{4x^3 - 12x^2 - 180x + 31}$$

So, $f_2(x)$ is also decreasing and positive in $\{-3, 0\}$

$\therefore$ absolute maximum value of $f(x)$ occurs at $x = -3$

$\therefore \qquad \alpha = -3$

Q.7 The curve $y(x) = ax^3 + bx^2 + cx + 5$ touches the x-axis at the point $P(-2, 0)$ and cuts the y-axis at the point Q, where y' is equal to 3. Then the local maximum value of $y(x)$ is:

(a) $\frac{27}{4}$ (b) $\frac{29}{4}$

(c) $\frac{37}{4}$ (d) $\frac{9}{2}$

7. (a)

$$f(x) = y = ax^3 + bx^2 + cx + 5 \qquad ...(i)$$

$$\frac{dy}{dx} = 3ax^2 + 2bx + c \qquad ...(ii)$$

Touches x-axis at P(–2, 0)

$$\Rightarrow \qquad y\big|_{x=-2} = 0 \Rightarrow -8a + 4b - 2c + 5 = 0 \qquad ...(iii)$$

Touches x-axis at $P(-2, 0)$ also implies

$$\left.\frac{dy}{dx}\right|_{x=-2} = 0 \Rightarrow 12a - 4b + c = 0 \qquad ...(iv)$$

$$y = f(x) \text{ cuts y-axis at } (0, 5)$$

Given,

$$\left.\frac{dy}{dx}\right|_{x=0} = c = 3 \qquad ...(v)$$

From (*iii*), (*iv*) and (*v*)

$$a = -\frac{1}{2}, b = -\frac{3}{4}, c = 3$$

$\Rightarrow$
$$f(x) = \frac{-x^2}{2} - \frac{3}{4}x^2 + 3x + 5$$

$$f'(x) = \frac{-3}{2}x^2 - \frac{3}{2}x + 3$$

$$= \frac{-3}{2}(x+2)(x-1)$$

$$f(x) = 0 \text{ at } x = -2 \text{ and } x = 1$$

Local maximum value of $f(x)$ is at $x = 1$

i.e., $\frac{27}{4}$

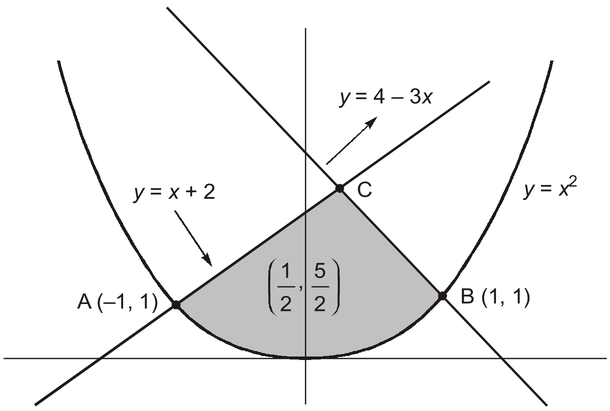
Q.8 The area of the region given by

$A = [(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}]$ is :

(a) $\frac{31}{8}$ (b) $\frac{17}{6}$

(c) $\frac{19}{6}$ (d) $\frac{27}{8}$

8. (b)



$A = [(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}$

So, area of the required region

$$A = \int_{-1}^{\frac{1}{2}} \left(x + 2 - x^2\right)dx + \int_{\frac{1}{2}}^{1} \left(4 - 3x - x^2\right)dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{\frac{1}{2}} + \left[4x - \frac{3x^2}{2} - \frac{x^3}{3}\right]_{\frac{1}{2}}^{1}$$

$$= \left(\frac{1}{8} + 1 - \frac{1}{24}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) + \left(4 - \frac{3}{2} + \frac{1}{3}\right) - \left(2 - \frac{3}{8} - \frac{1}{24}\right) = \frac{17}{6}$$

Q.9 For any real number x, let $[x]$ denote the largest integer less than equal to x. Let f be a real valued function defined on the interval $[-10, 10]$ by $f(x) = \begin{cases} x-[x], \text{if } [x] \text{ is odd} \\ 1+[x]-x, \text{if } [x] \text{ is even} \end{cases}$. Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x \, dx$ is:

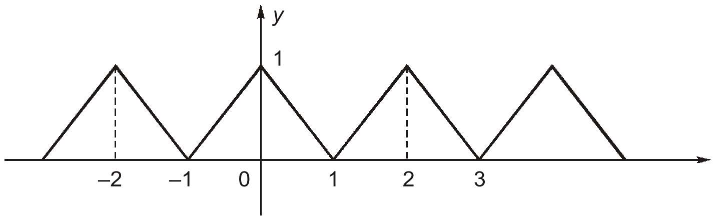
(a) 4 (b) 2

(c) 1 (d) 0

9. (a)

$$f(x) = \begin{cases} x-[x] \text{ if } [x] \text{ is odd} \\ 1+[x]-x, \text{if } [x] \text{ is even} \end{cases}$$

Graph of $f(x)$



So,

$$\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi \, dx = \frac{\pi^2}{10}\cdot 20\int_0^1 f(x)\cos\pi x \, dx$$

$$= 2\pi^2\int_0^1 (1-x)\cos\pi x \, dx$$

$$= 2\pi^2 \left\{ (1-x)\frac{\sin \pi x}{\pi}\Big|_0^1 - \frac{\cos \pi x}{\pi^2}\Big|_0^1 \right\} = 4$$

Q.10 The slope of the tangent to a curve $C : y = y(x)$ at any point (x, y) on it is $\frac{2e^{2x} - 6e^{-x} + 9}{2 + 9e^{-2x}}$. If C passes through the points $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$, then e^{α} is equal to :

(a) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$ (b) $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

(c) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ (d) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

10. (b)

$$\frac{dy}{dx} = \frac{2e^{2x} - 6e^{-x} + 9}{2 + 9e^{-2x}} = e^{2x} - \frac{6e^{-x}}{2 + 9e^{-2x}}$$

$$\int dy = \int e^{2x} dx - 3\int \frac{e^{-z}}{\underbrace{1 + \left(\frac{3e^{-x}}{\sqrt{2}}\right)}_{\text{put } e^{-x} = t}} dx$$

$$= \frac{e^{2x}}{2} + 3\int \frac{dt}{1+\left(\frac{3t}{\sqrt{2}}\right)^2}$$

$$= \frac{e^{2x}}{2} + \sqrt{2}\tan^{-1}\frac{3t}{\sqrt{2}} + C$$

$$y = \frac{e^{2x}}{2} + \sqrt{2}\tan^{-1}\left(\frac{3e^{-x}}{\sqrt{2}}\right) + C$$

It is given that the curve passes through

$$\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$$

$$\frac{1}{2} + \frac{\pi}{2\sqrt{2}} = \frac{1}{2} + \sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right) + C$$

$$\Rightarrow \quad C = \frac{\pi}{2\sqrt{2}} - \sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$$

Now if $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$

satisfies the curve, then

$$\frac{1}{2}e^{2\alpha} = \frac{e^{2\alpha}}{2} + \sqrt{2}\tan^{-1}\left(\frac{3e^{-\alpha}}{\sqrt{2}}\right) + \frac{\pi}{2\sqrt{2}} - \sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$$

$$\tan^{-1}\left(\frac{3}{\sqrt{2}}\right) - \tan^{-1}\left(\frac{3e^{-\alpha}}{\sqrt{2}}\right) = \frac{\pi}{2\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

$$\frac{\frac{3}{\sqrt{2}} - \frac{3e^{-\alpha}}{\sqrt{2}}}{1+\frac{9}{2}e^{-\alpha}} = 1$$

$$\frac{3}{\sqrt{2}}e^{\alpha} - \frac{3}{\sqrt{2}} = e^{\alpha} + \frac{9}{2}$$

$$e^{\alpha} = \frac{\frac{9}{2} + \frac{3}{\sqrt{2}}}{\frac{3}{\sqrt{2}} - 1} = \frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$$

Q.11 The general solution of the differential equation $(x - y^2)\,dx + y\,(5x + y^2)\,dy = 0$ is :

(a) $(y^2 + x)^4 = C|(y^2 + 2x)^3|$
(b) $(y^2 + 2x)^4 = C|(y^2 + x)^3|$
(c) $|(y^2 + x)^3| = C\,(2y^2 + x)^4$
(d) $|(y^2 + 2x)^3| = C\,(2y^2 + x)^4$

11. (a)

$$(x - y^2)\,dx + y\,(5x + y^2)\,dy = 0$$

$$y\frac{dy}{dx} = \frac{y^2 - x}{5x + y^2}$$

Let $y^2 = t$

$$\frac{1}{2}\cdot\frac{dt}{dx} = \frac{t - x}{5x + t}$$

Now substitute, $t = vx$

$$\frac{dt}{dx} = v + x\frac{dv}{dx}$$

$$\frac{1}{2}\left\{v + x\frac{dv}{dx}\right\} = \frac{v-1}{5+v}$$

$$x\frac{dv}{dx} = \frac{2v-2}{5+v} - v = \frac{-3v - v^2 - 2}{5+v}$$

$$\int \frac{5+v}{v^2+3v+2}dv = \int -\frac{dx}{x}$$

$$\int \frac{4}{v+1}dv - \int \frac{3}{v+2}dv = -\int \frac{dx}{x}$$

$$4\ \ln|v+1| - 3\ \ln|v+2| = -\ln x + \ln C$$

$$\left|\frac{(v+1)^4}{(v+2)^3}\right| = \frac{C}{x}$$

$$\left|\frac{\left(\frac{y^2}{x}+1\right)^4}{\left(\frac{y^2}{x}+2\right)^3}\right| = \frac{C}{x}$$

$$\left|\left(y^2+x\right)^4\right| = C\left|\left(y^2+2x\right)^3\right|$$

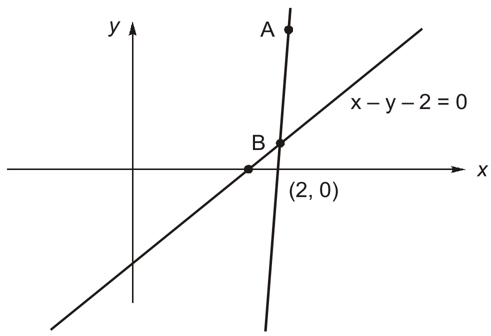
Q.12 A line, with the slope greater than one, passes through the point $A(4, 3)$ and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line :

(a) $2x + y = 9$ (b) $3x - 2y = 7$
(c) $x + 2y = 6$ (d) $2x - 3y = 3$

12. (c)

Let inclination of required line is θ,
So the coordinates of point B can be assumed as

$$\left(4 - \frac{\sqrt{29}}{3}\cos\theta, 3 - \frac{\sqrt{29}}{3}\sin\theta\right)$$



Which satisfies $x - y - 2 = 0$

$$4 - \frac{\sqrt{29}}{3}\cos\theta - 3 + \frac{\sqrt{29}}{3}\sin\theta - 2 = 0$$

$$\sin\theta - \cos\theta = \frac{3}{\sqrt{29}}$$

By squaring

$$\sin\theta - \cos\theta = \frac{20}{29} = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\tan\theta = \frac{5}{2} \text{ only (because slope is greater than 1)}$$

$$\sin\theta = \frac{5}{\sqrt{29}}, \cos\theta = \frac{2}{\sqrt{29}}$$

Point B : $\left(\frac{10}{3}, \frac{4}{3}\right)$

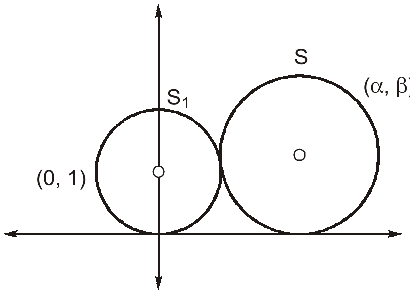
Which also satisfies $x + 2y = 6$

Q.13 Let the locus of the centre (α, β), $\beta > 0$, of the circle which touches the circle $x^2 + (y-1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line $y = 4$ is :

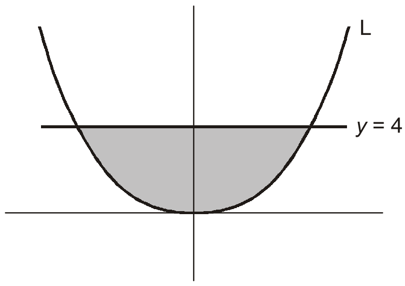
(a) $\frac{32\sqrt{2}}{3}$ (b) $\frac{40\sqrt{2}}{3}$

(c) $\frac{64}{3}$ (d) $\frac{32}{3}$

13. (c)



Redius of circle S touching x-axis and centre (α, β) is $|\beta|$. According to given conditions



$$\alpha^2 + (\beta - 1)^2 = (|\beta| + 1)^2$$

$$\alpha^2 + \beta^2 - 2\beta + 1 = \beta^2 + 1 + 2|\beta|$$

$$\alpha^2 = 4\beta \text{ as } \beta > 0$$

$\therefore$ Required locus is $L : x^2 = 4y$

The area of shaded region $= 2\int_0^4 2\sqrt{y}dy$

$$= 4 \cdot \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$

$$= \frac{64}{3} \text{ square units.}$$

Q.14 Let P be the plane containing the straight line $\frac{x-3}{9} = \frac{y+4}{-1} = \frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$. If d is the distance of P from the point $(2, -5, 11)$, then d^2 is equal to :

(a) $\frac{147}{2}$ (b) 96

(c) $\frac{32}{3}$ (d) 54

14. (c)

Let a, b, c be direction ratios of plane containing lines

$\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$

and

$\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$

$\therefore \quad 2a + 3b + 5c = 0 \quad (i)$

and $\quad 3a + 7b + 8c = 0 \quad (ii)$

From eq. (*i*) and (*ii*)

$$\frac{a}{24-35} = \frac{b}{15-16} = \frac{c}{14-9}$$

$\therefore$ D.R. of plane P be $< a_1, b_1, c_1 >$ then.

$$11a_1 + b_1 - 5c_1 = 0 \quad (iii)$$

and $\quad 9a_1 - b_1 - 5c_1 = 0 \quad (iv)$

From eq. (iii) and (iv) :

$$\frac{a_1}{-5-5} = \frac{b_1}{-45+55} = \frac{c_1}{-11-9}$$

Equation plane P is : $1(x-3) - 1(y+4) + 2(z-7) = 0$

$\Rightarrow \quad x - y + 2z - 21 = 0$

Distance from point $(2, -5, 11)$ is

$$d = \frac{|2+5+22-2|}{\sqrt{6}}$$

$\therefore \quad d^2 = \frac{32}{3}$

Q.15 Let ABC be a triangle such that $\overrightarrow{BC} = \vec{a}, \overrightarrow{CA} = \vec{b}, \overrightarrow{AB} = \vec{c}, |\vec{a}| = 6\sqrt{2}, |\vec{b}| = 2\sqrt{3}$ and $\vec{b}\cdot\vec{c} = 12$. Consider the statements:

(S1) : $|(\vec{a}\times\vec{b}) + (\vec{c}\times\vec{b})| - |\vec{c}| = 6(2\sqrt{2} - 1)$

(S2) : $\angle ACB = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Then

(a) both (S1) and S2) are true
(b) only (S1) is true
(c) only (S2) is true
(d) both (S1) and S2) are false

15. (c)

$\because \quad \vec{a} + \vec{b} + \vec{c} = 0 \quad ...(i)$

then

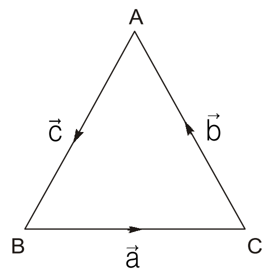
$$\vec{a} + \vec{c} = -\vec{b}$$

then

$$(\vec{a} + \vec{c}) \times \vec{b} = -\vec{b} \times \vec{b}$$

$\therefore \quad \vec{a}\times\vec{b} + \vec{c}\times\vec{b} = \vec{0}$

Now (S1) : $\quad |\vec{a}\times\vec{b} + \vec{c}\times\vec{b}| - |\vec{c}| = 6(2\sqrt{2} - 1)$



$$\left|(\vec{a}+\vec{c})\times\vec{b}\right| - |\vec{c}| = 6(2\sqrt{2}-1)$$
$$|\vec{c}| = 6-12\sqrt{2}$$

Hence (S1) is not correct
For (S2) : from (*i*)

$$\vec{b}+\vec{c} = -\vec{a}$$

$\Rightarrow$ $$\vec{b}\cdot\vec{b}+\vec{c}\cdot\vec{b} = -\vec{a}\cdot\vec{b}$$

$\Rightarrow$ $$12+12 = -6\sqrt{2}\cdot 2\sqrt{3}\cos(\pi-\angle ACB)$$

$\therefore$ $$\cos(\angle ACB) = \sqrt{\frac{2}{3}}$$

$\therefore$ $$\angle ACB = \cos^{-1}\sqrt{\frac{2}{3}}$$

$\therefore$ S(2) is correct.

Q.16 If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is :

(a) $\frac{33}{2^{32}}$ (b) $\frac{33}{2^{29}}$

(c) $\frac{33}{2^{28}}$ (d) $\frac{33}{2^{27}}$

16. (c)

If n is number of trials, p is probability of success and q is probability of unsuccess then,

Mean = np and variance = npq.

Here $$np + npq = 24 \quad ...(i)$$
$$np\cdot npq = 128 \quad ...(ii)$$
and $$q = 1-p \quad ...(iii)$$

From eq. (*i*), (*ii*) and (*iii*) :

$$p = q = \frac{1}{2}$$

and $$n = 32$$

$\therefore$ Required probability

$$= P(X=1) + P(X-2)$$
$$= {}^{32}C_1\cdot\left(\frac{1}{2}\right)^{32} + {}^{32}C_2\cdot\left(\frac{1}{2}\right)^{32}$$
$$= \left(32+\frac{32\times 31}{2}\right)\cdot\frac{1}{2^{32}}$$
$$= \frac{33}{2^{28}}$$

Q.17 If the numbers appeared on the two throws of fair six faced die are α and β, then the probability that $x^2+\alpha x+\beta > 0$, for all $x \in R$, is :

(a) $\frac{17}{36}$ (b) $\frac{4}{9}$

(c) $\frac{1}{2}$ (d) $\frac{19}{36}$

17. (a)

For $x^2 + \alpha x + \beta > 0 \ \forall \ x \in R$ to hold, we should have $\alpha^2 - 4\beta < 0$
If $\alpha = 1$, β can be 1, 2, 3, 4, 5, 6 i.e., 6 choices
If $\alpha = 2$, β can be 2, 3, 4, 5, 6 i.e., 5 choices
If $\alpha = 3$, β can be 3, 4, 5, 6 i.e., 4 choices
If $\alpha = 4$, β can be 5 or 6 i.e., 2 choices
If $\alpha = 6$, No possible value for β i.e., 0 choices
Hence total favourable outcomes
$= 6 + 5 + 4 + 2 + 0 + 0 = 17$
Total possible choices for α and $\beta = 6 \times 6 = 36$

Required probability $= \dfrac{17}{36}$

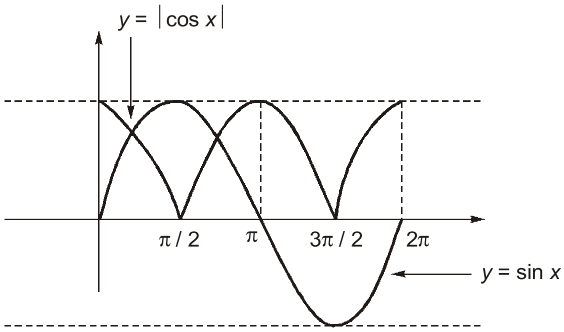
Q.18 The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is :

(a) 4 (b) 6
(c) 8 (d) 12

18. (c)

Number of solutions of the equation $|\cos x| = \sin x$ for $x \in [-4\pi, 4\pi]$ will be equal to 4 times the number of solutions of the same equation for $x \in [0, 2\pi]$

Graphs of $y = |\cos x|$ and $y = \sin x$ are as shown below.



Hence, two solutions of given equation in $[0, 2\pi]$
$\Rightarrow$ Total of 8 solutions in $[-4\pi, 4\pi]$

Q.19 A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that $QR = 15$ m. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is :

(a) $5(2\sqrt{3}+3)$m (b) $5(\sqrt{3}+3)$m
(c) $10(\sqrt{3}+1)$m (d) $10(2\sqrt{3}+1)$m

19. (a)

From ΔAPQ

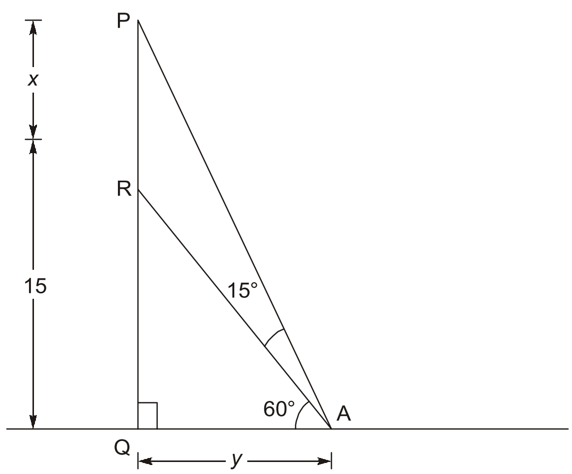
$$\frac{x+15}{y} = \tan 75° \qquad ...(i)$$

From ΔRQA

$$\frac{15}{y} = \tan 60° \qquad ...(ii)$$

From (*i*) and (*ii*)

$$\frac{x+15}{15} = \frac{\tan 75°}{\tan 60°} = \frac{\tan(45° + 30°)}{\tan 60°} = \frac{\sqrt{3}+1}{(\sqrt{3}-1)\cdot\sqrt{3}}$$



On simplification,

$$x = 10\sqrt{3} \text{ cm}$$
$$= (15 + 10\sqrt{3}) \text{ m}$$
$$= 5(2\sqrt{3} + 3) \text{ m}$$

Q.20 Which of the following statements is a tautology?

(a) $((\sim p) \vee q) \Rightarrow p$ (b) $p \Rightarrow ((\sim p) \vee q)$

(c) $((\sim p) \vee q) \Rightarrow q$ (d) $q \Rightarrow ((\sim p) \vee q)$

20. (d)

Truth Table

p	q	~p	~q	(~p) ∨ q	((~p) ∨ q) → p	p → ((~p) ∨ q)	(~p) ∨ q → q	q → ((~p) ∨ q)
T	T	F	F	T	T	T	T	T
T	F	F	T	F	T	F	T	T
F	T	T	F	T	F	T	T	T
F	F	T	T	T	F	T	F	T

SECTION - B
(Numerical Answer Type)

Q.1 Let $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$ and $B = A - I$. If $\omega = \frac{\sqrt{3}i - 1}{2}$, then the number of elements in the set $\{n \in \{1, 2, ..., 100\} : A^n + (\omega B)^n = A + B\}$ is equal to ....

1. (17)

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

We get $A^2 = A$ and similarly, for

$$B = A - I = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

We get $B^2 = -B \Rightarrow B^3 = B$

$\therefore$ $A^n + (\omega B)^n = A + (\omega B)^n$ for $n \in N$

For ω^n to be unity n shall be multiple of 3 and for Bn to be B. n shell be 3,5,7, ...9

$\therefore$ $n = \{3, 9, 15, ...99\}$

Number of elements = 17.

Q.2 The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is............ .

2. (1492)

A............ →

$\frac{6!}{2!} = 360$

D........... $\rightarrow$ 360

I .... $\rightarrow$ 360

K........... $\rightarrow$ 360

$M\ A\ D$............ $\rightarrow$

$\frac{4!}{2!} = 12$

$M\ A\ I$............ $\rightarrow$ 12

$M\ A\ K$............ $\rightarrow$ 12

$M\ A\ N\ D$............ $\rightarrow 3! = 6$

$M\ A\ N\ I$............. $\rightarrow$ 6

$M\ A\ N\ K\ D$............ $\rightarrow$ 2

$M\ A\ N\ K\ I\ D$............ $\rightarrow$ 1

$M\ A\ N\ K\ I\ N\ D$............ $\rightarrow$ 1

$\therefore$ Rank of MANKIND = 1440 + 36 + 12 + 2 + 2 = 1492

Q.3 If the maximum value of the term independent of t in the expansion of $\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \geq 0$, is K, then 8 K is equal to.............

3. **(6006)**

General term

$$= {}^{15}C_r \left(t^2 x^{\frac{1}{5}}\right)^{15-r} \left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^r$$

For term independent on t

$$2(15 - r) - r = 0$$

$$\Rightarrow \quad r = 10$$

$$\therefore \quad T_{11} = {}^{15}C_{10} \times (1 - x)$$

Maximum value of $x(1 - x)$ occur at

$$x = \frac{1}{2}$$

$$\text{i.e., } (x(1 - x))_{max} = \frac{1}{4}$$

$$\Rightarrow \quad K = {}^{15}C_{10} \times \frac{1}{4}$$

$$\Rightarrow \quad 8K = 2({}^{15}C_{10}) = 6006$$

Q.4 Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to ....

4. **(38)**

∵ Roots of $2ax^2 - 8ax + 1 = 0$ are

$\frac{1}{P}$ and $\frac{1}{r}$ and roots of $6bx^2 + 12bx + 1 = 0$ are

$\frac{1}{q}$ and $\frac{1}{8}$

Let $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{8}$

as $\alpha - 3\beta$, $\alpha - \beta$, $\alpha + \beta$, $\alpha + 3\beta$

So sum of roots $2\alpha - 2\beta = 4$ and $2\alpha + 2\beta = -2$

Clearly

$$\alpha = \frac{1}{2}$$

and

$$\beta = -\frac{3}{2}$$

Now product of roots,

$$\frac{1}{p}\cdot\frac{1}{r} = \frac{1}{2a} = -5 \Rightarrow \frac{1}{a} = -10$$

and

$$\frac{1}{q}\cdot\frac{1}{x} = \frac{1}{6a} = -8 \Rightarrow \frac{1}{b} = -48$$

So, $$\frac{1}{a} - \frac{1}{b} = 38$$

Q.5 Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to ....

5. (27560)

$$a_1 = B_1 = 1$$
$$a_n = a_{n-1} + 2 \text{ (for } n \geq 2) \; b_n = a_n + b_{n-1}$$
$$a_2 = a_1 + 2 = 1 + 2 = 3 \; b_2 = a_2 + b_1 = 3 + 1 = 4$$
$$a_3 = a_2 + 2 = 3 + 2 = 5 \; b_3 = a_3 + b_2 = 5 + 4 = 9$$

Similarly for others

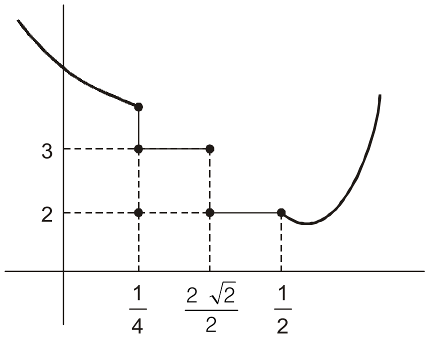
$$\sum_{n=1}^{11} a_n b_n = \sum_{n=1}^{15}(2n-1)n^2 = \sum_{n=1}^{15} 2n^3 - \sum_{n=1}^{15} n^2$$
$$= 2\left[\frac{15\times16}{2}\right]^2 - \left[\frac{15\times16\times31}{6}\right] = 27560$$

Q.6 Let $f(x) = \begin{cases} \left|4x^2 - 8x + 5\right|, \text{if } 8x^2 - 6x + 1 \geq 0 \\ \left[4x^2 - 8x + 5\right], \text{if } 8x^2 - 6x + 1 < 0 \end{cases}$, where $[\alpha]$ denotes the greatest integer less than or equal to α. Then the number of points in R where f is not differentiable is ....

6. (3)

$$f(x) = \begin{cases} |4x^2 - 8x + 5|, \text{if } 8x^2 - 6x + 1 \geq 0 \\ [4x^2 - 8x + 5], \text{if } 8x^2 - 6x + 1 < 0 \end{cases}$$

$$= \begin{cases} 4x^2 - 8x + 5, & \text{if } x \in \left[-\infty, \frac{1}{4}\right] \cup \left[\frac{1}{2}, \infty\right] \\ [4x^2 - 8x + 5], & \text{if } x \in \left(\frac{1}{4}, \frac{1}{2}\right) \end{cases}$$



$$f(x) = \begin{cases} 4x^2 - 8x + 5 & x \in \left(-\infty, \frac{1}{4}\right] \cup \left[\frac{1}{2}, \infty\right) \\ 3, & x \in \left(\frac{1}{4}, \frac{2-\sqrt{2}}{2}\right) \\ 2, & x \in \left[\frac{2-\sqrt{2}}{2}, \frac{1}{2}\right) \end{cases}$$

$\therefore$ Non-diff at

$$x = \frac{1}{4}, \frac{2-\sqrt{2}}{2}, \frac{1}{2}$$

Q.7 If $\lim\limits_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}}$ $[(nk + 1) + (nk + 2) + ... + (nk + n)]$

$= 33. \lim\limits_{n\to\infty} \frac{1}{n^{k+1}}$ $[1^k + 2^k + 3^k + ... + n^k]$, then the integral value of k is equal to ....

7. (5)

$$\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^{k-1} \frac{1}{n}\sum_{r=1}^{n}\left(k + \frac{r}{n}\right)$$

$$= 33\int_0^1 x^k dx$$

Q.8 Let the equation of two diameters of a circle $x^2 + y^2 - 2x + 2fy + 1 = 0$ be $2px - y = 1$ and $2x + py = 4p$. Then the slope $m \in (0,\infty)$ of the tangent to the hyperbola $3x^2 - y^2 = 3$ passing through the centre of the circle is equal to................. .

8. (2)

$$x^2 + y^2 - 2x + 2fy + 1 = 0 \text{ centre} = (1, -f)$$

Diameter $2px - y = 1$ ...(*i*)

$2x + py = 4p$ ...(*ii*)

$$x = \frac{5P}{2P^2 + 2}$$

$$y = \frac{4P^2 - 1}{1 + P^2}$$

$\because$ $x = 1$

$f = 0$

$$\left[\text{for } P = \frac{1}{2}\right]$$

$$\frac{5P}{2P^2+2} = 1$$

$$f = 3 \text{ [for } P = 2]$$

$$\therefore \quad P = \frac{1}{2}, 2$$

Centre can be $\left(\frac{1}{2}, 0\right)$ or (1, 3)

$\left(\frac{1}{2}, 0\right)$ will not satisfy

$\therefore$ Tangent should pass through (2, 3) for $3x^2 - y^2 = 3$

$$\frac{x^2}{1} - \frac{y^2}{3} = 1$$

$$y = mx \pm \sqrt{m^2 - 3}$$

Substitute (2, 3)

$$3 = m \pm \sqrt{m^2 - 3}$$

$$\therefore \quad m = 2$$

Q.9 The sum of diameters of the circles that touch (*i*) the parabola $75x^2 = 64(5y - 3)$ at the point $\left(\frac{8}{5}, \frac{6}{5}\right)$ and (ii), the y-axis, is equal to ....

9. **(10)**

$P\left(\frac{8}{5}, \frac{6}{5}\right)$

$$75x \cdot \frac{8}{5} = 160\left(y + \frac{6}{5}\right) - 192$$

$$\Rightarrow \quad 120\,x = 160\,y$$

$$\Rightarrow \quad 3\,x = 4\,y$$

Equation of circle touching the given parabola at P can be taken as

$$\left(x - \frac{8}{5}\right)^2 + \left(y - \frac{6}{5}\right)^2 + \lambda(3x - 4y) = 0$$

In this circule touching y-axis then

$$\frac{64}{25} + \left(y - \frac{6}{5}\right)^2 + \lambda(-4y) = 0$$

$$\Rightarrow \quad y^2 - 2y\left(2\lambda + \frac{6}{5}\right) + 4 = 0$$

$$\Rightarrow \quad D = 0$$

$$\Rightarrow \quad \left(2\lambda + \frac{6}{6}\right)^2 = 4$$

$$\Rightarrow \quad \lambda = \frac{2}{5} \text{ or } -\frac{8}{5}$$

Redius = 1 or 4

Sum of diameter = 10

Q.10 The line of shortest distance between the lines $\frac{x-2}{0}=\frac{y-1}{1}=\frac{z}{1}$ and $\frac{x-3}{2}=\frac{y-5}{2}=\frac{z-1}{1}$ makes an angle of $\cos^{-1}\left(\sqrt{\frac{2}{27}}\right)$ with the plane $P: ax-y-z=0, (a>0)$. If the image of the point $(1, 1, -5)$ in the plane P is (α, β, λ), then $\alpha+\beta-\gamma$ is equal to ....

10. (DROP)

Line of shortest distance will be along $\overline{b_1}\times\overline{b_2}$

Where,

$$\overline{b_1} = \hat{j}+\hat{k}$$

and

$$\overrightarrow{b_2} = 2\hat{i}+2\hat{j}+\hat{k}$$

$$\overline{b_1}\times\overline{b_2} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 1\\ 2 & 2 & 1\end{vmatrix} = -\hat{i}+2\hat{j}-2\hat{k}$$

Angle between $\overline{b_1}\times\overline{b_2}$ and plane P.

$$\sin\theta = \left|\frac{-a-2+2}{3\cdot\sqrt{a^2+2}}\right| = \frac{5}{\sqrt{27}} \Rightarrow \frac{|a|}{\sqrt{a^2+2}} = \frac{5}{\sqrt{3}}$$

$\Rightarrow$ $$a^2 = -\frac{25}{11} \text{ (not possible)}$$